NTNU | Norwegian University of Science and Technology

Bayesian seismic rock physics inversion using a localized ensemble-based approach - with an application to the Alvheim field

Mina Spremic[1], Jo Eidsvik[1] and Per Avseth[1,2]
[1]Norwegian University of Science and Technology (NTNU)
[2]Dig Science

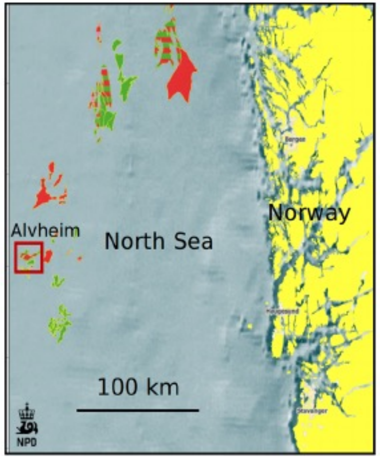
Outline

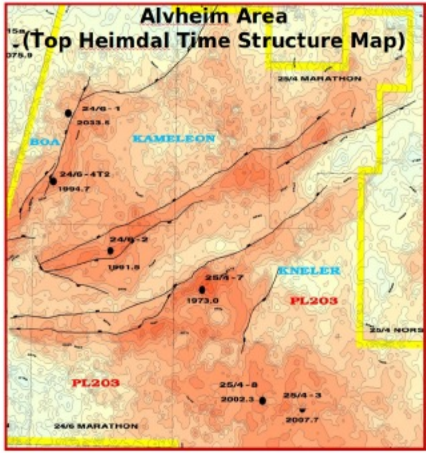
- Goals
- Case: Alvheim field
- Likelihood and geophysical forward model
- Prior model
- Posterior approximation
- Concluding remarks and further work

Goals

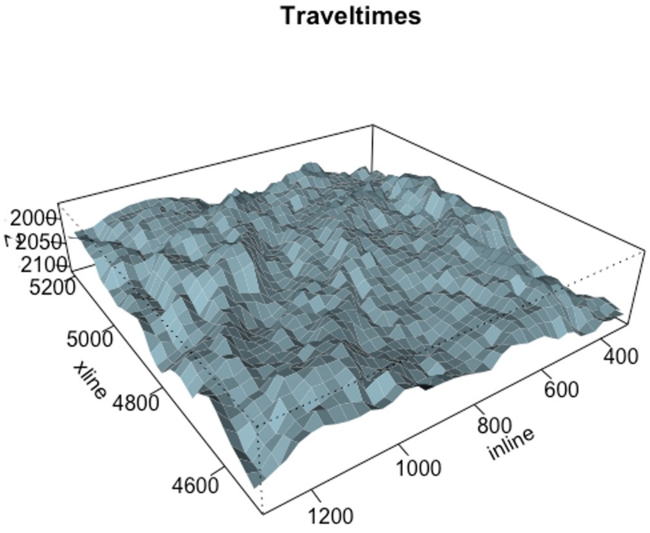
- Invert for the continuous reservoir parameters: oil and gas saturations and clay content
- Perform 2.5D inversion on top-reservoir using localised ensemble methods
- Condition on the seismic and well log data
- Provide more realistic uncertainty quantification by accounting for cementation and geology of the area

Case: Alvheim field





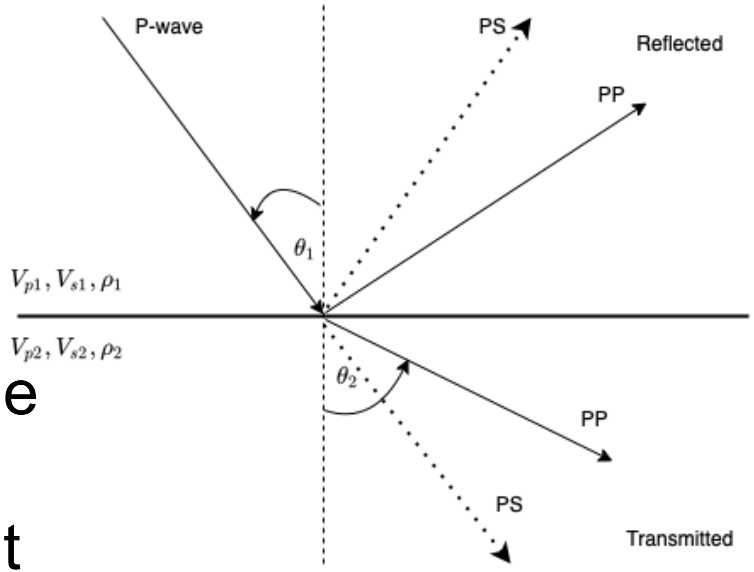
Source: Rimstad et al. *Hierarchical Bayesian lithology/fluid prediction: A North Sea case study*



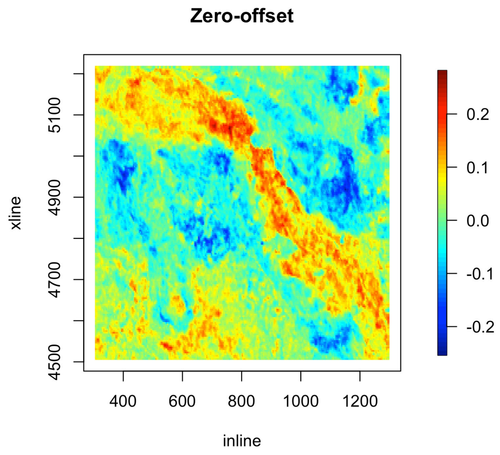
Traveltimes, top-reservoir

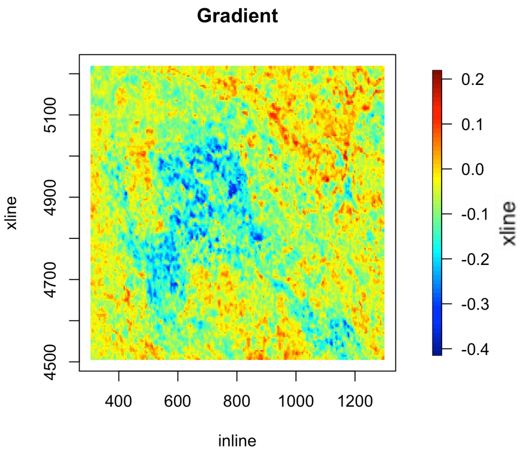
Amplitude versus offset - AVO

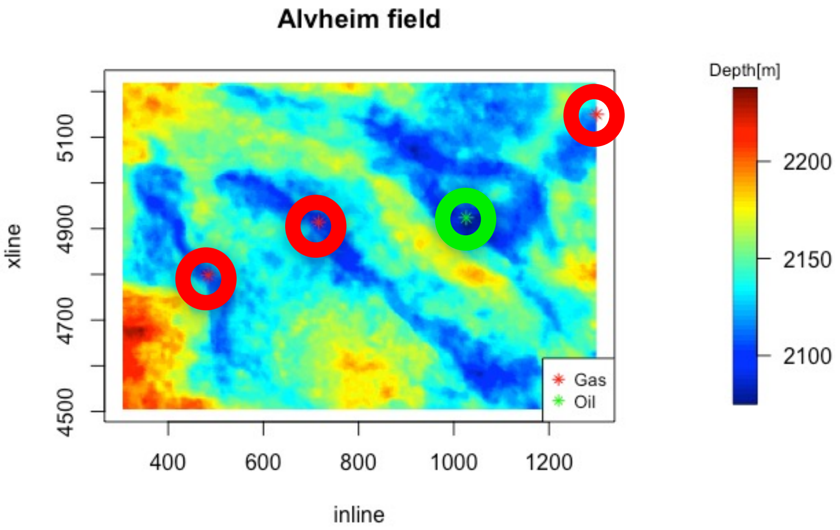
- Amplitude variation dependent on physical properties at the interface
- Zoeppritz equations
- Shuey approximation
- Linear fit of changes in amplitude vs incidence angles
- Extracting intercept and gradient at top-reservoir



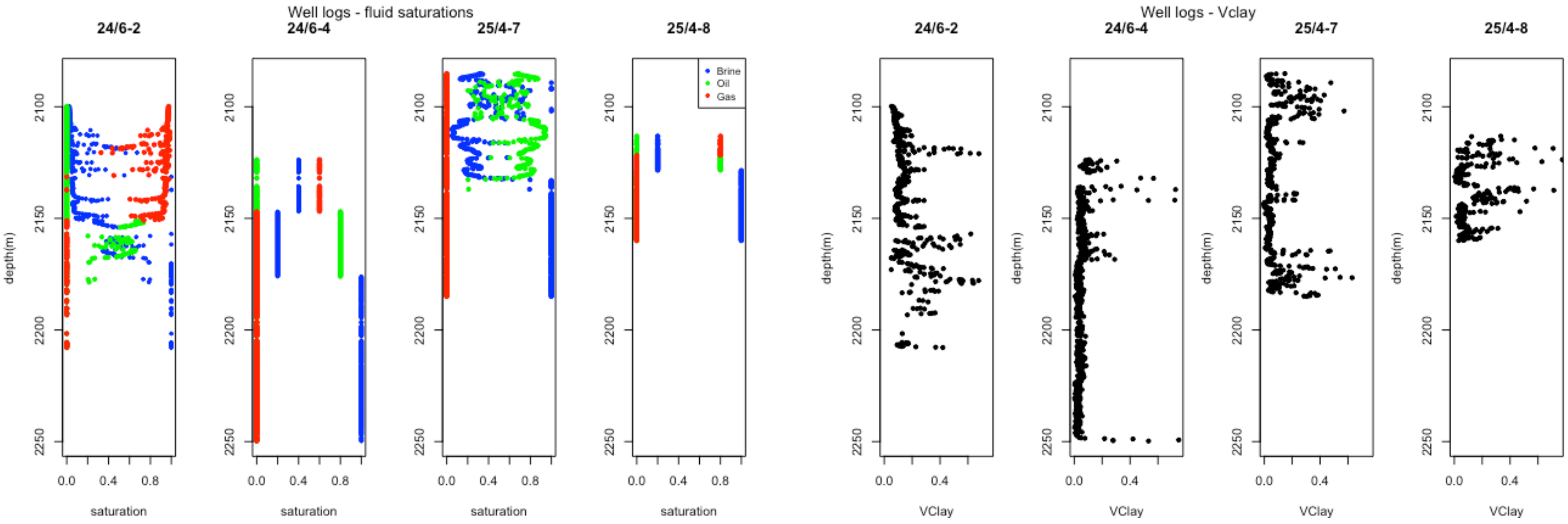
Case: Alvheim AVO data at top-reservoir







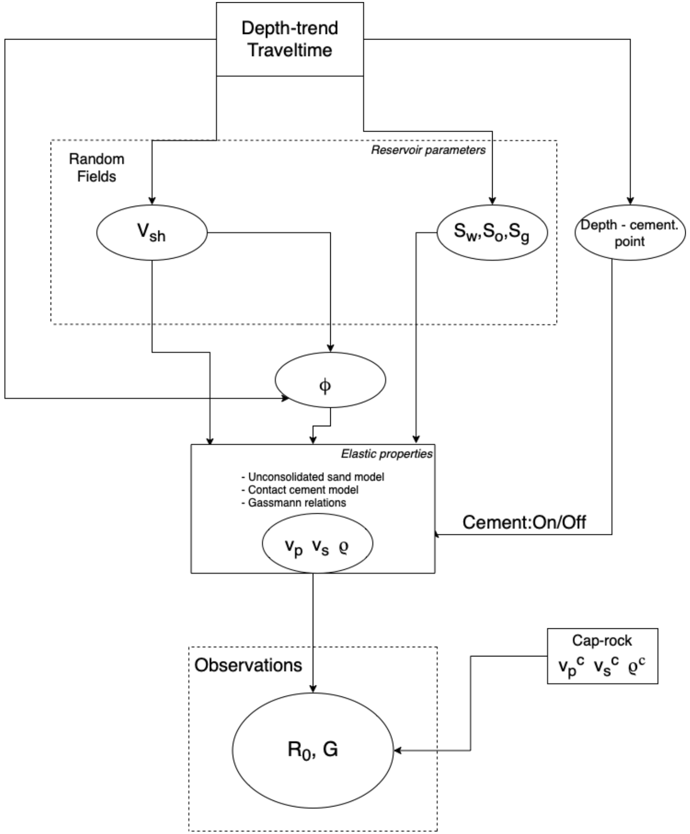
Case: Alvheim well log data



Geophysical model

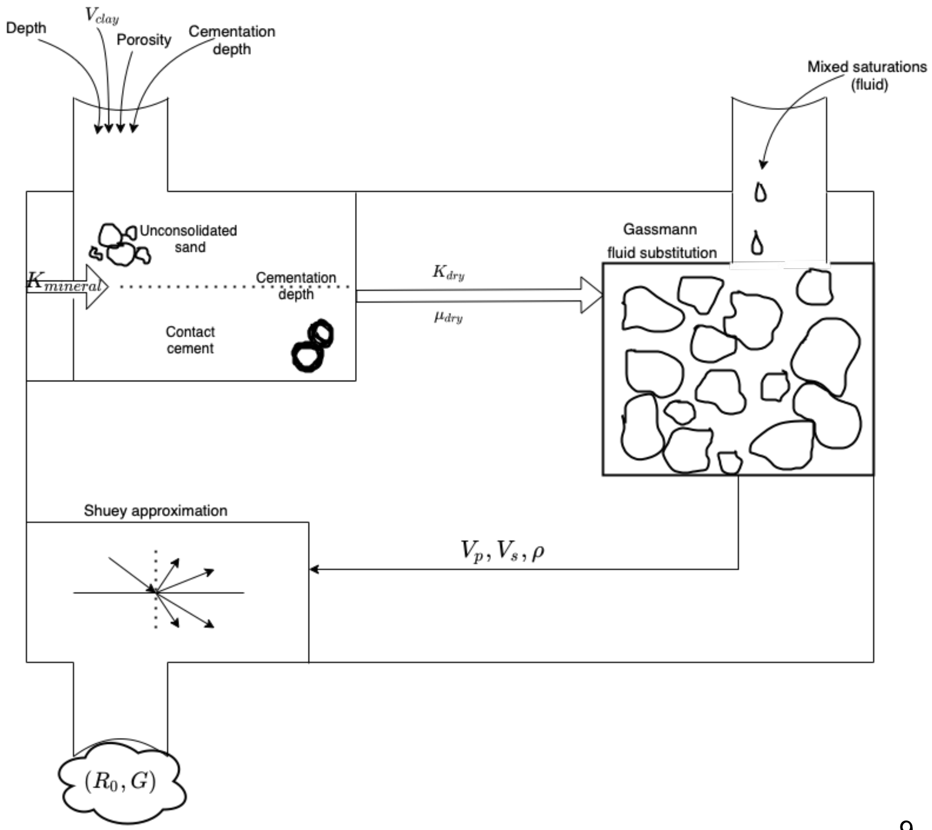
Prior model

Forward model and likelihood



Zooming in on the seismic forward model

- Forward model
  - Unconsolidated sand model
  - Contact cement model (Dvorkin-Nur)
  - Gassmann relations
  - Shuey approximation



Bayesian framework

- Bayes rule

$$p(\boldsymbol{x}|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{x})p(\boldsymbol{x})$$

- Prior model

$$p(\boldsymbol{x}^{\boldsymbol{e}}) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

- Likelihood
  - Observation model: $\mathbf{y}^{\boldsymbol{e}} = \boldsymbol{h}(\boldsymbol{x}^{\boldsymbol{e}}) + \boldsymbol{\epsilon}$
  - Non-linear forward model; dependent on depth, cementation point, porosity
  - $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \boldsymbol{R})$, $\boldsymbol{R}$- block diagonal

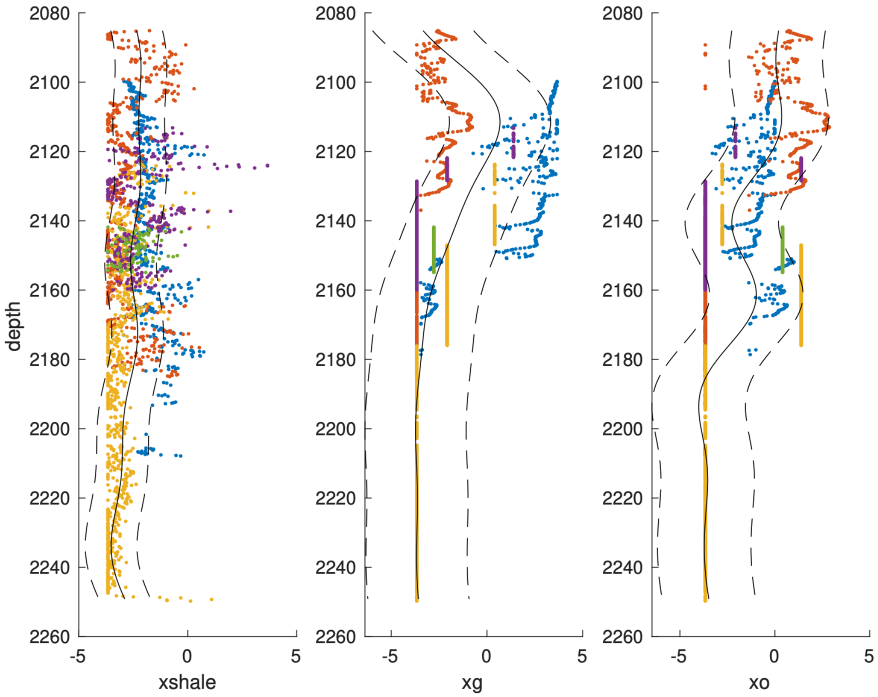
Prior model

- Gaussian prior

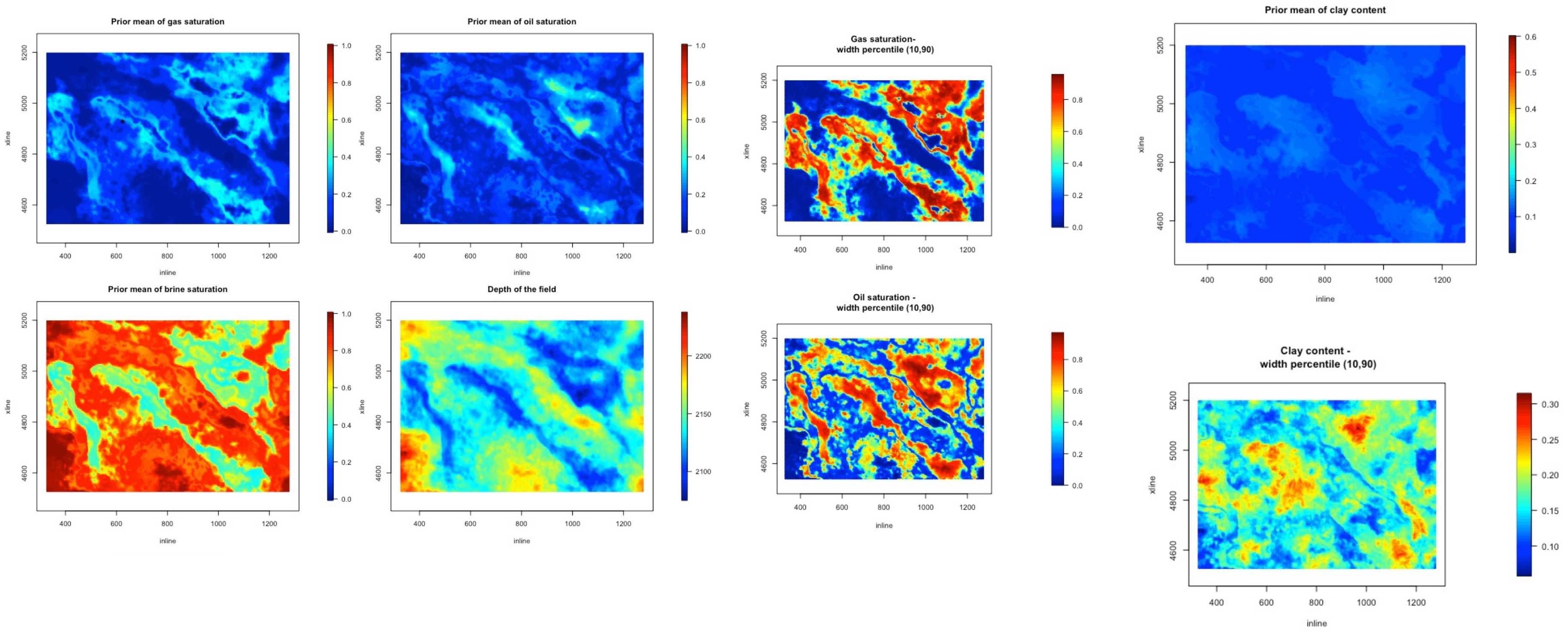
$$\boldsymbol{x}^g \sim N(\boldsymbol{\mu}^g, \boldsymbol{\Sigma}^g)$$
$$\boldsymbol{x}^0 \sim N(\boldsymbol{\mu}^o, \boldsymbol{\Sigma}^o)$$
$$\boldsymbol{x}^{cl} \sim N(\boldsymbol{\mu}^{cl}, \boldsymbol{\Sigma}^{cl})$$

- Mean trends estimated from well logs (depth dependence)
- Gaussian spatial correlation function
- Ensembles generated using the FFT-routine
- Logistic transformation

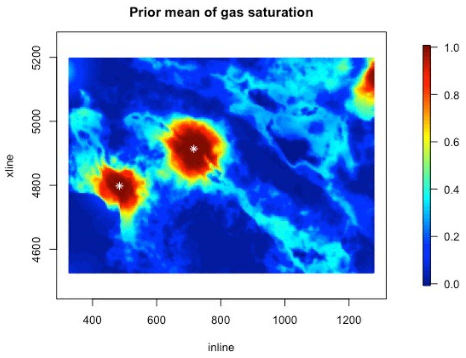
$$S_g = \frac{\exp(x^g)}{1 + \exp(x^g) + \exp(x^o)}$$

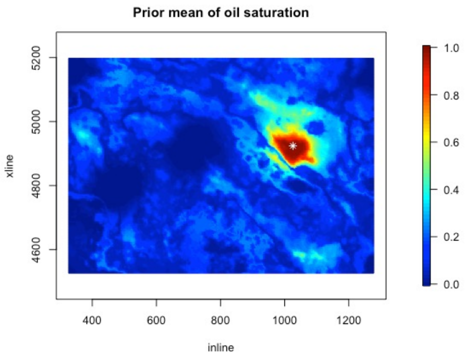


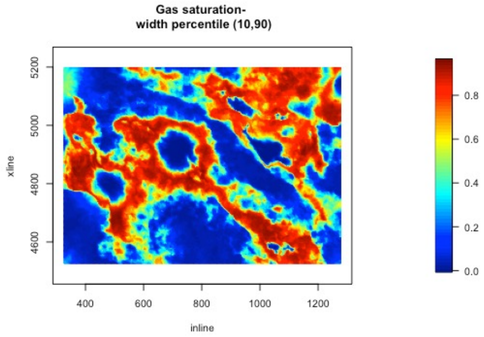
Prior: mean and uncertainty from realisations

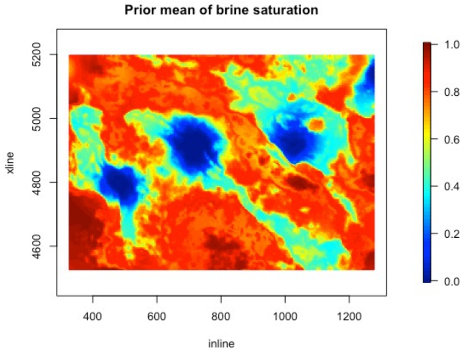


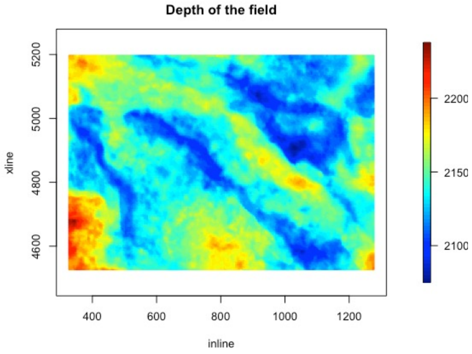
Conditioning on well data

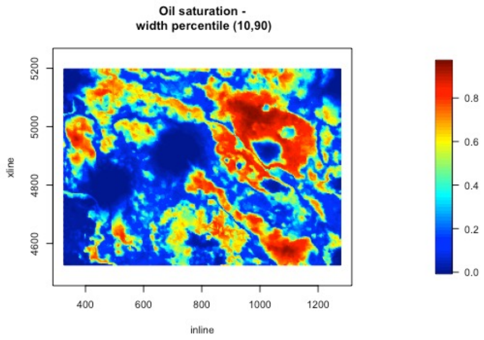






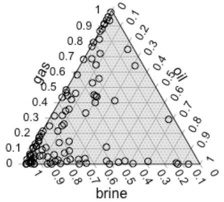




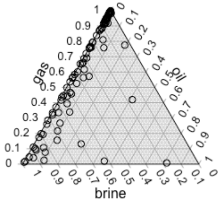


Conditioning on well data

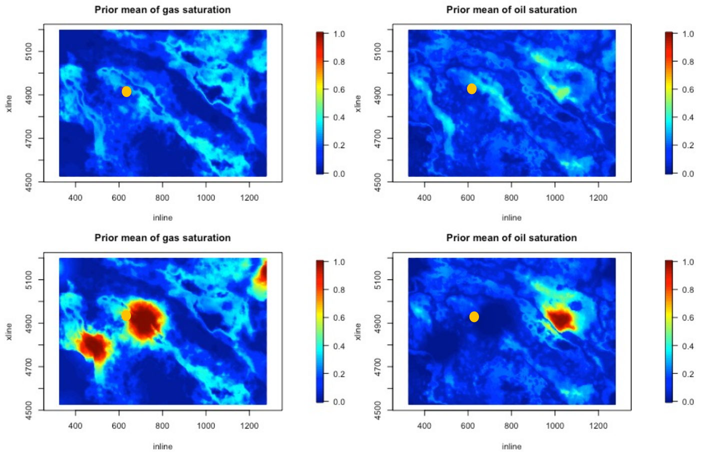
- Difference in saturation at inline 624, xline 4928 for 100 realisations



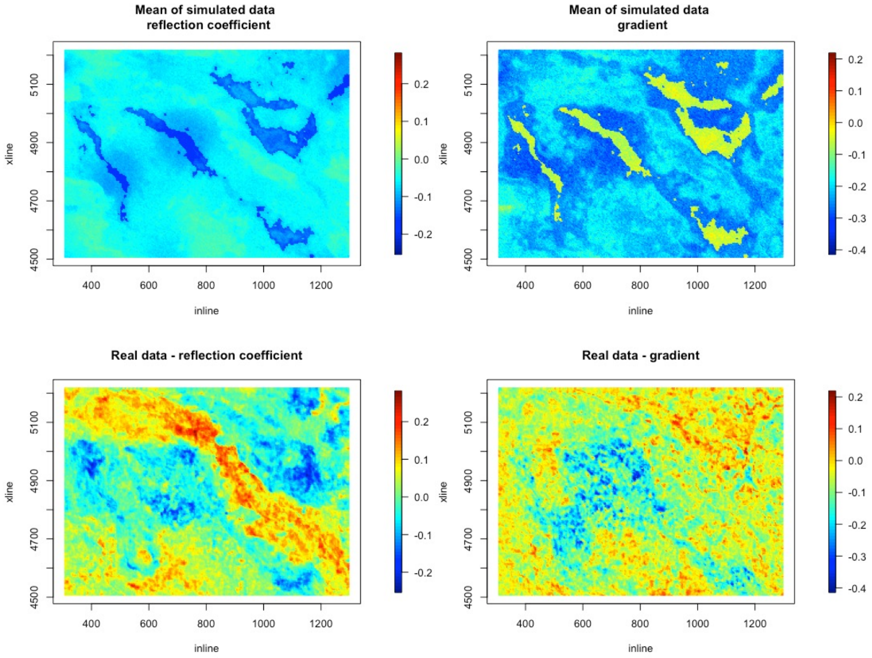
unconditioned

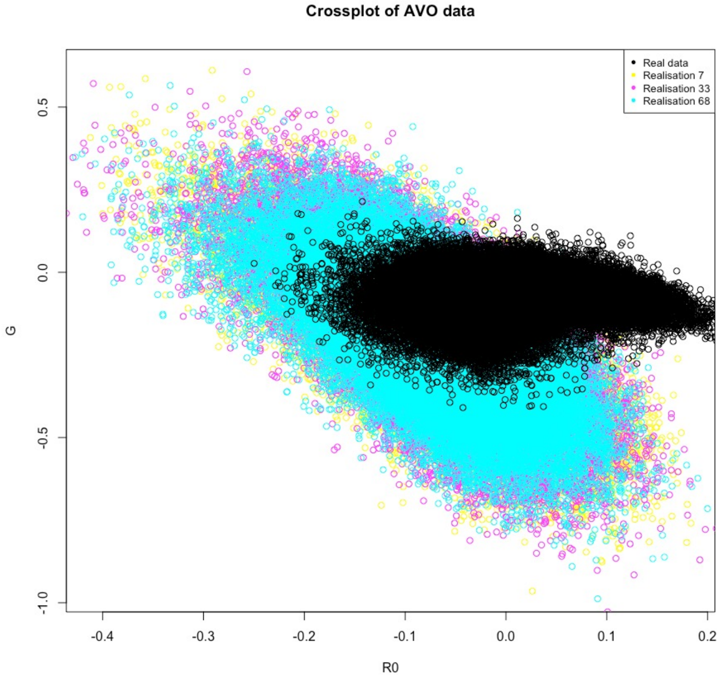


conditioned



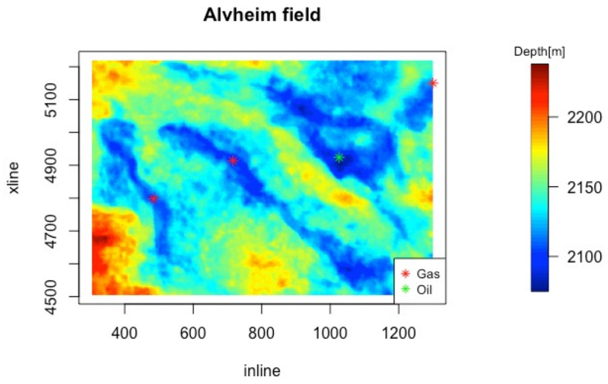
Simulated seismic data





Posterior approximation- EnKF

- Approximate realisations of the posterior
- Localised Ensemble Transform Kalman Filter



Algorithm 1: Localized Ensemble Transform Kalman Filter

Prior

$\boldsymbol{x}^e \sim p(\boldsymbol{x}) = N(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}), \quad e = 1, ..., n_e;$

$\boldsymbol{y}^e = h(\boldsymbol{x}^e) + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim N(\boldsymbol{\epsilon}; 0, \boldsymbol{R});$

$\bar{y}_n = 1/n_e \sum_{e=1}^{n_e} y_n^e, \quad n = 1, ..., N;$

$\boldsymbol{Y} = (\boldsymbol{y}^1 - \bar{\boldsymbol{y}}, ..., \boldsymbol{y}^{n_e} - \bar{\boldsymbol{y}});$

$\bar{x}_n = 1/n_e \sum_{e=1}^{n_e} x_n^e;$

$\boldsymbol{X} = (\boldsymbol{x}^1 - \bar{\boldsymbol{x}}, ..., \boldsymbol{x}^{n_e} - \bar{\boldsymbol{x}});$

For each patch $j = 1, ..., J$

$\boldsymbol{H}_j = [\boldsymbol{Y}_j^T \boldsymbol{R}_j^{-1} \boldsymbol{Y}_j + (n_e - 1)\boldsymbol{I}]^{-1}$

$\boldsymbol{K}_j = \boldsymbol{X}_j \boldsymbol{H}_j \boldsymbol{Y}_j^T \boldsymbol{R}_j^{-1}$

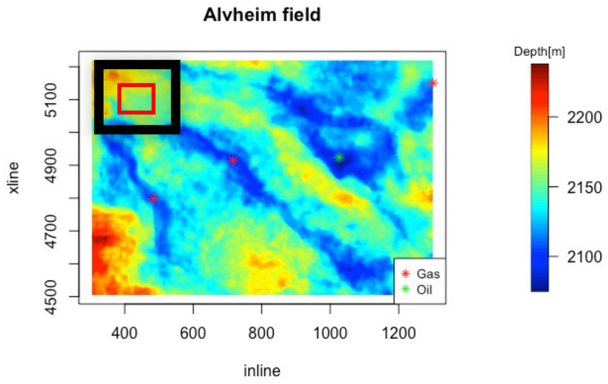
$\bar{\boldsymbol{x}}^a = \bar{\boldsymbol{x}} + \boldsymbol{K}_j(\boldsymbol{y}_j^0 - \bar{\boldsymbol{y}})$

$\boldsymbol{E}_j = \bar{\boldsymbol{x}}^a + \boldsymbol{X}_j[(n_e - 1)\boldsymbol{H}_j]^{1/2};$

Result: $\boldsymbol{E} = (\boldsymbol{E}_1, \ldots, \boldsymbol{E}_j)$

Posterior approximation- EnKF

- Approximate realisations of the posterior
- Localised Ensemble Transform Kalman Filter



Algorithm 1: Localized Ensemble Transform Kalman Filter

Prior

$\boldsymbol{x}^e \sim p(\boldsymbol{x}) = N(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}), \quad e = 1, ..., n_e;$

$\boldsymbol{y}^e = h(\boldsymbol{x}^e) + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim N(\boldsymbol{\epsilon}; 0, \boldsymbol{R});$

$\bar{y}_n = 1/n_e \sum_{e=1}^{n_e} y_n^e, \quad n = 1, ..., N;$

$\boldsymbol{Y} = (\boldsymbol{y}^1 - \bar{\boldsymbol{y}}, ..., \boldsymbol{y}^{n_e} - \bar{\boldsymbol{y}});$

$\bar{x}_n = 1/n_e \sum_{e=1}^{n_e} x_n^e;$

$\boldsymbol{X} = (\boldsymbol{x}^1 - \bar{\boldsymbol{x}}, ..., \boldsymbol{x}^{n_e} - \bar{\boldsymbol{x}});$

For each patch $j = 1, ..., J$

$\boldsymbol{H}_j = [\boldsymbol{Y}_j^T \boldsymbol{R}_j^{-1} \boldsymbol{Y}_j + (n_e - 1)\boldsymbol{I}]^{-1}$

$\boldsymbol{K}_j = \boldsymbol{X}_j \boldsymbol{H}_j \boldsymbol{Y}_j^T \boldsymbol{R}_j^{-1}$

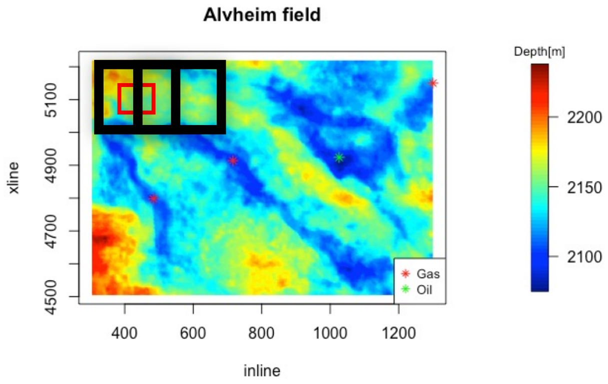
$\bar{\boldsymbol{x}}^a = \bar{\boldsymbol{x}} + \boldsymbol{K}_j(\boldsymbol{y}_j^0 - \bar{\boldsymbol{y}})$

$\boldsymbol{E}_j = \bar{\boldsymbol{x}}^a + \boldsymbol{X}_j[(n_e - 1)\boldsymbol{H}_j]^{1/2};$

Result: $\boldsymbol{E} = (\boldsymbol{E}_1, \ldots, \boldsymbol{E}_j)$

Posterior approximation- EnKF

- Approximate realisations of the posterior
- Localised Ensemble Transform Kalman Filter



Algorithm 1: Localized Ensemble Transform Kalman Filter

Prior

$\boldsymbol{x}^e \sim p(\boldsymbol{x}) = N(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}), \quad e = 1, ..., n_e;$

$\boldsymbol{y}^e = h(\boldsymbol{x}^e) + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim N(\boldsymbol{\epsilon}; 0, \boldsymbol{R});$

$\bar{y}_n = 1/n_e \sum_{e=1}^{n_e} y_n^e, \quad n = 1, ..., N;$

$\boldsymbol{Y} = (\boldsymbol{y}^1 - \bar{\boldsymbol{y}}, ..., \boldsymbol{y}^{n_e} - \bar{\boldsymbol{y}});$

$\bar{x}_n = 1/n_e \sum_{e=1}^{n_e} x_n^e;$

$\boldsymbol{X} = (\boldsymbol{x}^1 - \bar{\boldsymbol{x}}, ..., \boldsymbol{x}^{n_e} - \bar{\boldsymbol{x}});$

For each patch $j = 1, ..., J$

$\boldsymbol{H}_j = [\boldsymbol{Y}_j^T \boldsymbol{R}_j^{-1} \boldsymbol{Y}_j + (n_e - 1)\boldsymbol{I}]^{-1}$

$\boldsymbol{K}_j = \boldsymbol{X}_j \boldsymbol{H}_j \boldsymbol{Y}_j^T \boldsymbol{R}_j^{-1}$

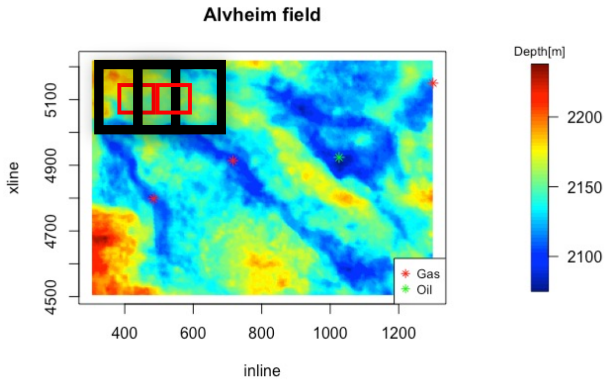
$\bar{\boldsymbol{x}}^a = \bar{\boldsymbol{x}} + \boldsymbol{K}_j(\boldsymbol{y}_j^0 - \bar{\boldsymbol{y}})$

$\boldsymbol{E}_j = \bar{\boldsymbol{x}}^a + \boldsymbol{X}_j[(n_e - 1)\boldsymbol{H}_j]^{1/2};$

Result: $\boldsymbol{E} = (\boldsymbol{E}_1, \ldots, \boldsymbol{E}_j)$

Posterior approximation- EnKF

- Approximate realisations of the posterior
- Localised Ensemble Transform Kalman Filter



Algorithm 1: Localized Ensemble Transform Kalman Filter

Prior

$\boldsymbol{x}^e \sim p(\boldsymbol{x}) = N(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}), \quad e = 1, ..., n_e;$

$\boldsymbol{y}^e = h(\boldsymbol{x}^e) + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim N(\boldsymbol{\epsilon}; 0, \boldsymbol{R});$

$\bar{y}_n = 1/n_e \sum_{e=1}^{n_e} y_n^e, \quad n = 1, ..., N;$

$\boldsymbol{Y} = (\boldsymbol{y}^1 - \bar{\boldsymbol{y}}, ..., \boldsymbol{y}^{n_e} - \bar{\boldsymbol{y}});$

$\bar{x}_n = 1/n_e \sum_{e=1}^{n_e} x_n^e;$

$\boldsymbol{X} = (\boldsymbol{x}^1 - \bar{\boldsymbol{x}}, ..., \boldsymbol{x}^{n_e} - \bar{\boldsymbol{x}});$

For each patch $j = 1, ..., J$

$\boldsymbol{H}_j = [\boldsymbol{Y}_j^T \boldsymbol{R}_j^{-1} \boldsymbol{Y}_j + (n_e - 1)\boldsymbol{I}]^{-1}$

$\boldsymbol{K}_j = \boldsymbol{X}_j \boldsymbol{H}_j \boldsymbol{Y}_j^T \boldsymbol{R}_j^{-1}$

$\bar{\boldsymbol{x}}^a = \bar{\boldsymbol{x}} + \boldsymbol{K}_j(\boldsymbol{y}_j^0 - \bar{\boldsymbol{y}})$

$\boldsymbol{E}_j = \bar{\boldsymbol{x}}^a + \boldsymbol{X}_j[(n_e - 1)\boldsymbol{H}_j]^{1/2};$

Result: $\boldsymbol{E} = (\boldsymbol{E}_1, \ldots, \boldsymbol{E}_j)$

Posterior approximation – IEnKS

- Localised IEnKS
- For each patch we run the IEnKS for *j* number of iterations
- Data assimilation window (DAW) corresponds to the observation patch

Algorithm 2: Localized Iterative Ensemble Kalman Smoother

Prior ensemble $\boldsymbol{E}^f = \boldsymbol{E}_k^f$

$j = 0,\ \boldsymbol{w}_j = \boldsymbol{0},\ \boldsymbol{T}_j = \boldsymbol{I}_{n_e}$

$\bar{\boldsymbol{x}}^f = \boldsymbol{E}^f \boldsymbol{1}/n_e$

$\boldsymbol{X}_f = \left(\boldsymbol{E}^f - \bar{\boldsymbol{x}}^f \boldsymbol{1}^T\right)/\sqrt{n_e - 1}$

Repeat

$\quad \boldsymbol{x}_j = \bar{\boldsymbol{x}}^f + \boldsymbol{X}_f \boldsymbol{w}_j$

$\quad \boldsymbol{E}_j = \boldsymbol{x}_j \boldsymbol{1}^T + \sqrt{n_e - 1}\boldsymbol{X}_f \boldsymbol{T}_j$

$\quad \bar{\boldsymbol{y}} = h_k(\boldsymbol{E}_j)\boldsymbol{1}/n_e$

$\quad \boldsymbol{Y} = \left[h_k(\boldsymbol{E}_j) - \bar{\boldsymbol{y}}\boldsymbol{1}^T\right]\boldsymbol{T}_j^{-1}/\sqrt{n_e - 1}$

$\quad \nabla J = \boldsymbol{w}_j - \boldsymbol{Y}^T \boldsymbol{R}_k^{-1}(\boldsymbol{y}_k^0 - \bar{\boldsymbol{y}})$

$\quad \mathbb{H} = \boldsymbol{I} + \boldsymbol{Y}^T \boldsymbol{R}_k^{-1} \boldsymbol{Y}$

$\quad \boldsymbol{w}_{j+1} = \boldsymbol{w}_j - \mathbb{H}^{-1}\nabla J$

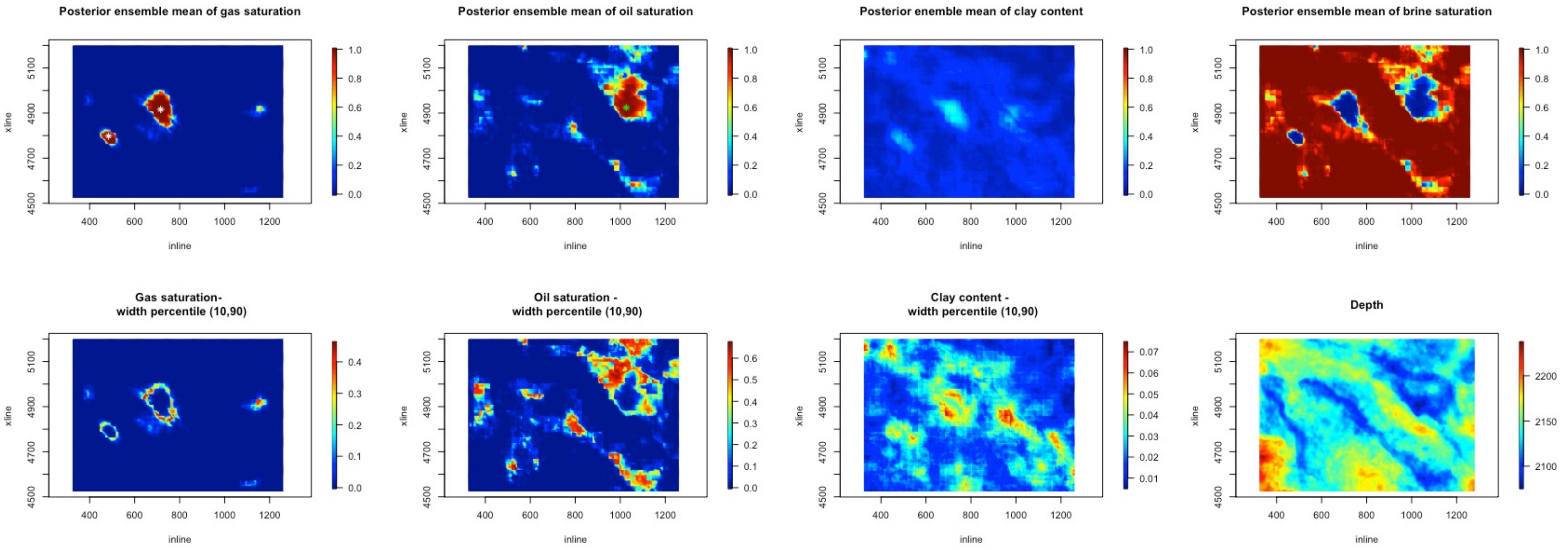
$\quad \boldsymbol{T}_{j+1} = \mathbb{H}^{-1/2}$

$\quad j = j + 1$

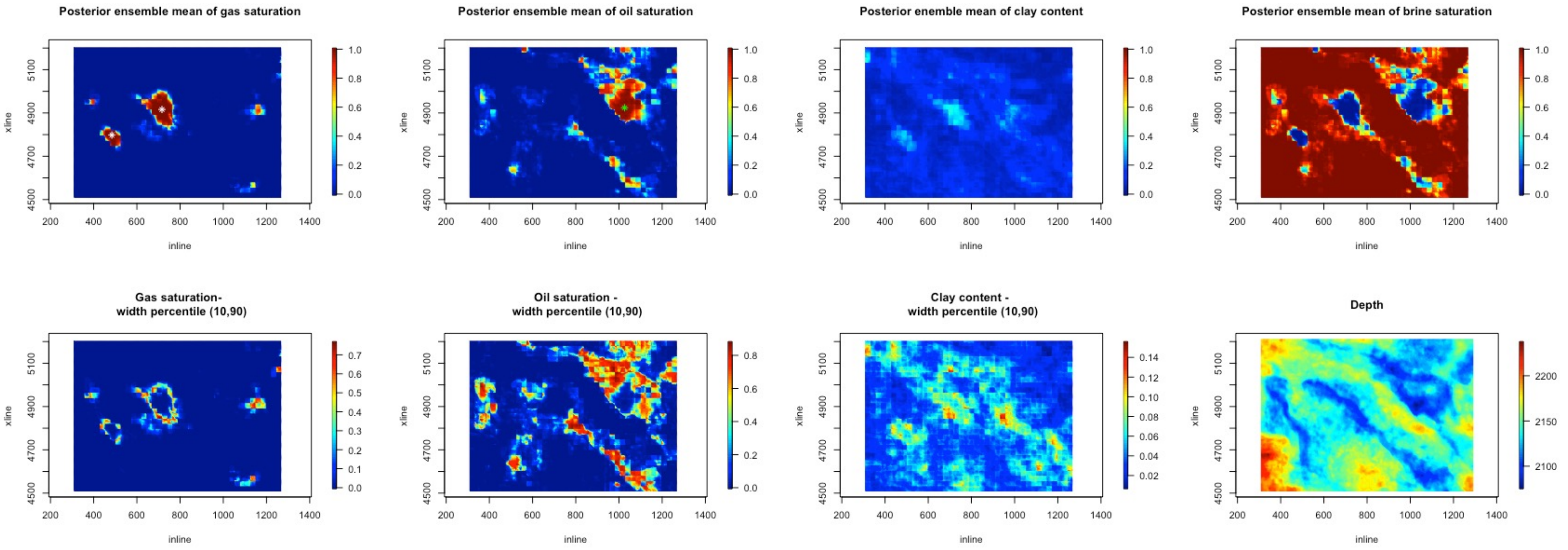
termination criteria met

$\boldsymbol{E}_k = \bar{\boldsymbol{x}}^f \boldsymbol{1}^T + \boldsymbol{X}_f\left(\boldsymbol{w}_{j-1}^T \boldsymbol{1}^T + \sqrt{n_e - 1}\boldsymbol{T}_j\right)$

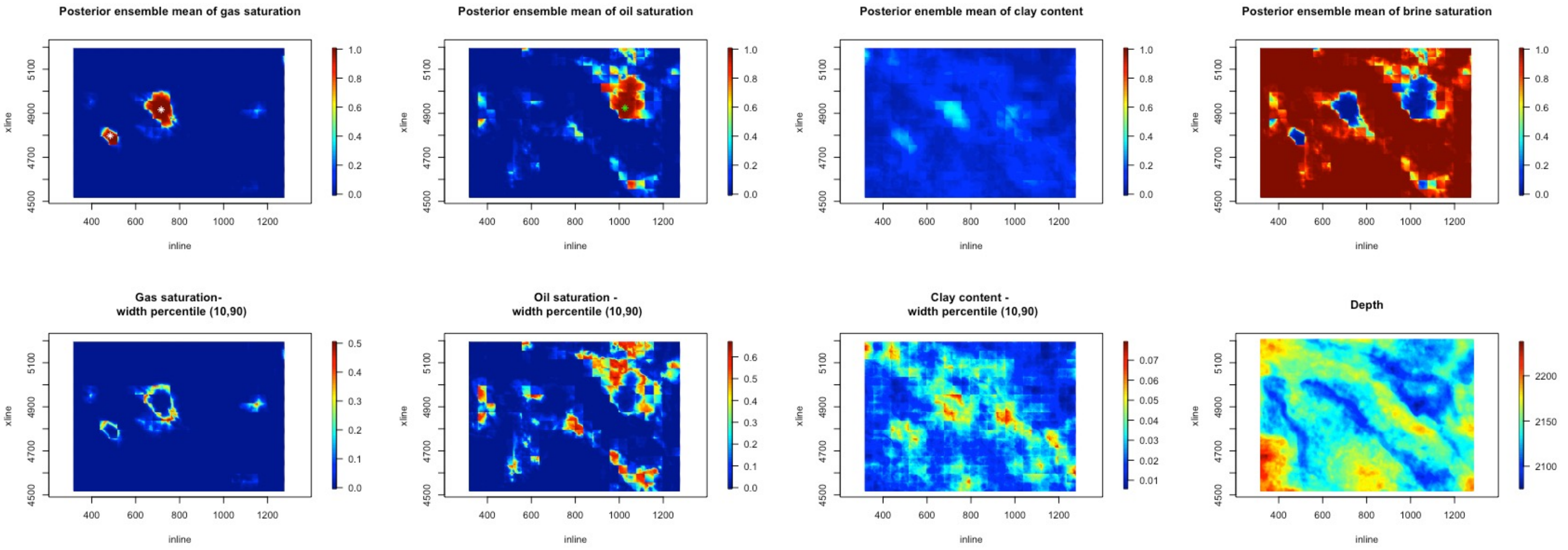
Posterior approximation – results(ref. case) Observation patch: 16x16; Parameters 6x6;



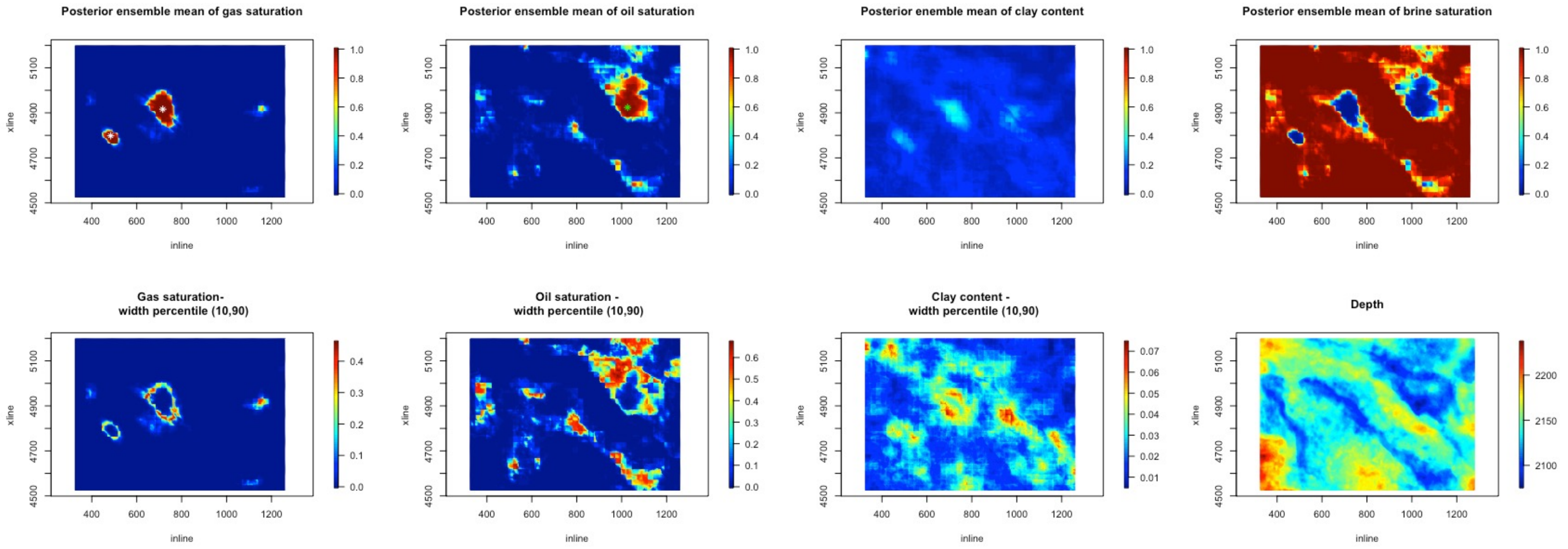
Posterior approximation – results
Observation patch: 9x9; Parameters: 6x6;



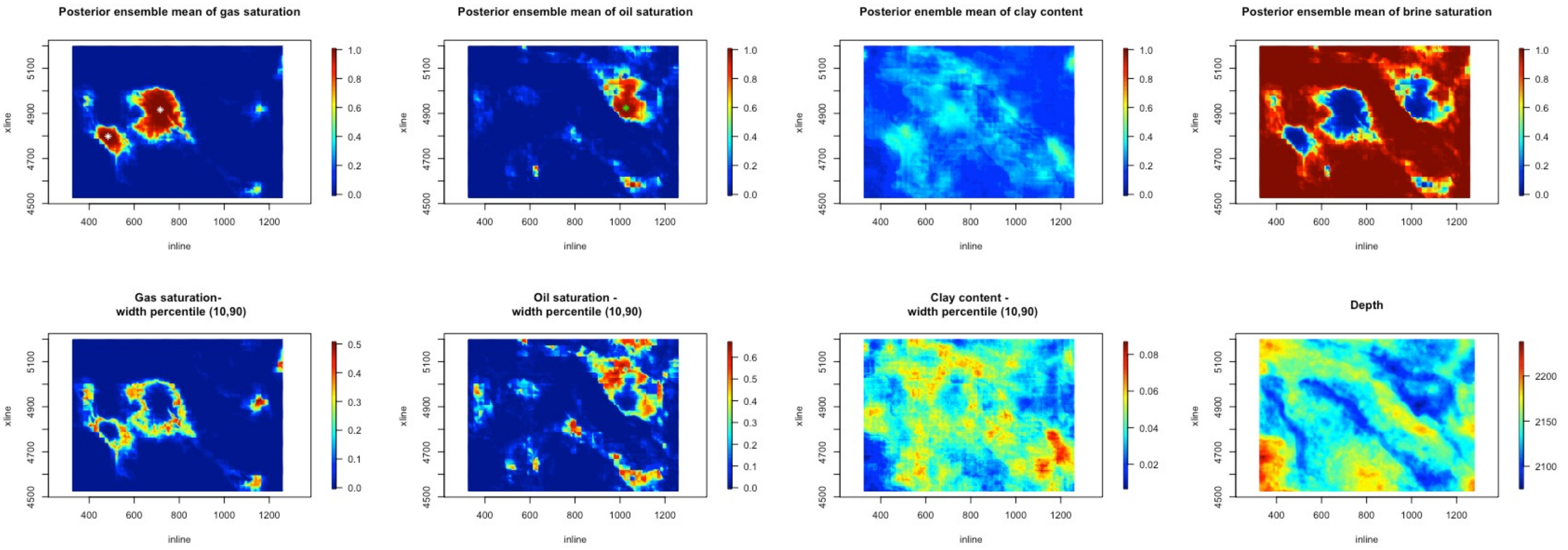
Posterior approximation – results
Observation patch: 16x16; Parameters: 10x10;



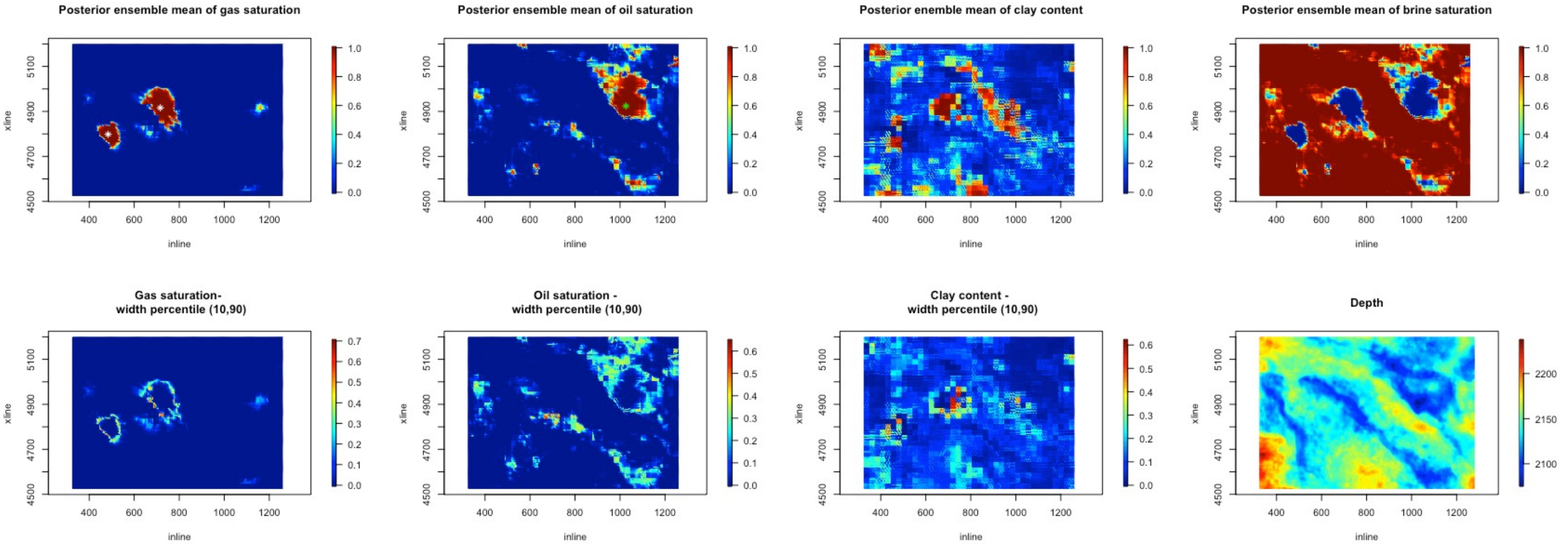
Posterior approximation – results
Cem depth: 2110 m



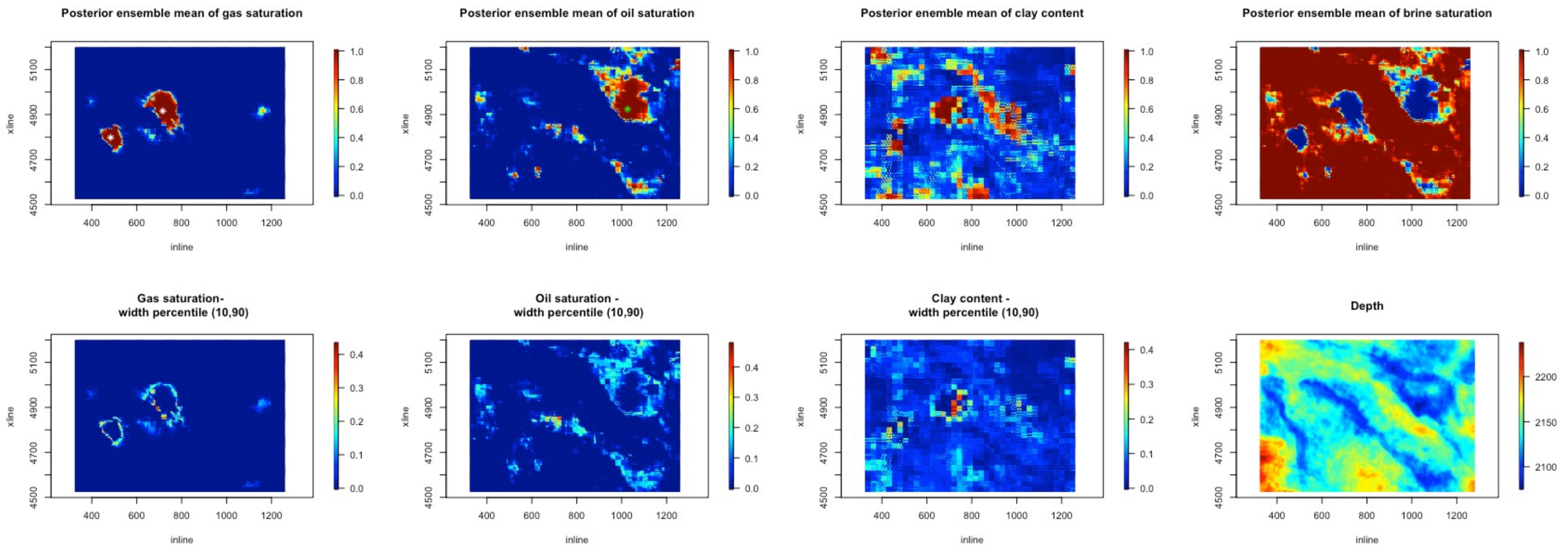
Posterior approximation – results
Cem depth: 2162 m



Posterior approximation – results
One iteration



Posterior approximation – results
Two iterations



Concluding remarks and further work

- Perform 2.5D inversion of the reservoir parameters
- Use localised ensemble methods to assimilate available seismic data

- Validation of the results using seismic data and sensitivity analysis
- Further improving forward model and conditioning on remaining well log data