A data assimilation approach for the estimation of mantle viscosities from paleo sea level observations

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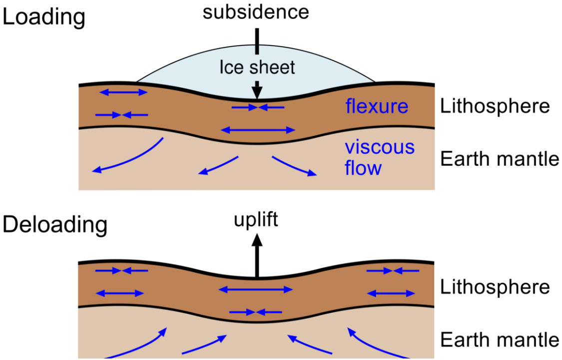
Overview

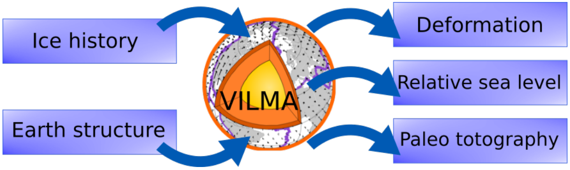
- Goal: estimation of **mantle viscosities**
- Assimilation of relative paleo sea level observations in the GIA model VILMA
- Sandbox experiment with observations taken from reference run (identical twin setup)
- Assimilation of **sea level rates of change**
- Two viscosity distribution parameterizations:
    1. 3-layer model with two viscous mantle layers and (fixed) elastic lithosphere
    2. 1D profile with 152 viscous mantle layers and (fixed) elastic lithosphere

Glacial Isostatic Adjustment



- Earth's visco-elastic response to changing mass load at the surface
- Involves lateral flow of mantle material
- Subsidence / uplift rates depend on mantle's ability to flow (viscosity)

VILMA



- Forward model for Earth's visco-elastic deformation due to glaciation / deglaciation (Klemann et al., 2008)
- Computes visco-elastic response of spherical Earth to surface mass load change
- Uses spectral finite-element approach (Martinec, 2000)
- Models deformation & solves sea-level equation to obtain relative sea levels

Data assimilation

- Use particle filter with stochastic resampling and perturbation
- During assimilation step particle performance is estimated based on observations
- Resampling of low-weight particles to model states of higher-weight particles
- Perturbation of particle viscosity values to
  - Avoid filter degeneracy
  - Explore new model state space

The particle filter

Bayes' Theorem for pdfs

$$p_m(\psi|d) = \frac{p_d(d|\psi)p_m(\psi)}{p_d(d)} \quad (Leeuwen, 2009)$$

where

$p_m(\psi\|d)$	posterior pdf for model given the data
$p_d(d\|\psi)$	likelihood of data given the model
$p_m(\psi)$	prior pdf of the model
$p_d(d)$	model evidence

Filter update

Posterior density:

$$p(\psi|d) = \sum_{i=1}^{N} w_i \delta(\psi - \psi_i)$$

with weights

$$w_i = \frac{p(d|\psi_i)}{\sum_{j=1}^{N} p(d|\psi_j)}$$

and likelihood

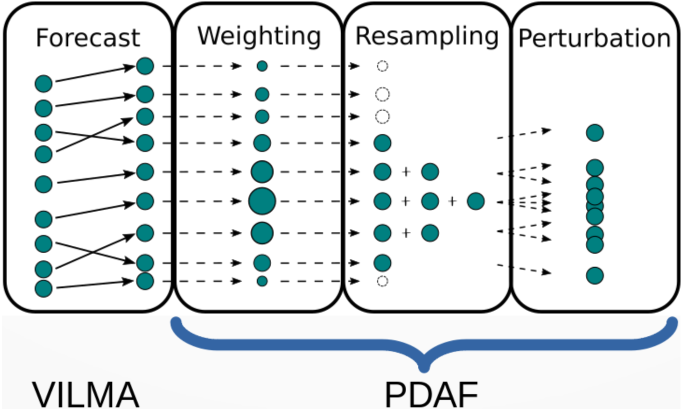
$$p(d|\psi_i) = \exp\left(\frac{1}{2\sigma_d^2}(d - \mathbf{H}\psi_i)^{\mathrm{T}}(d - \mathbf{H}\psi_i)\right)$$

with ψ_i: model vector, $\mathbf{H}$: observation operator, d: observation vector

3-layer case: perturbation based on ensemble variance: $\Delta\psi_i \sim N(0, \sigma_{\mathrm{ens}}^2)$

The particle filter

- Particle filter with resampling and perturbation
- Make use of Parallel Data Assimilation Framework PDAF (Nerger et al., 2005)



The particle filter

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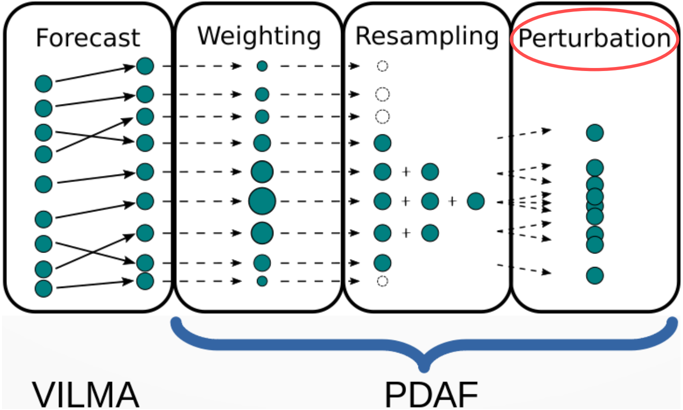
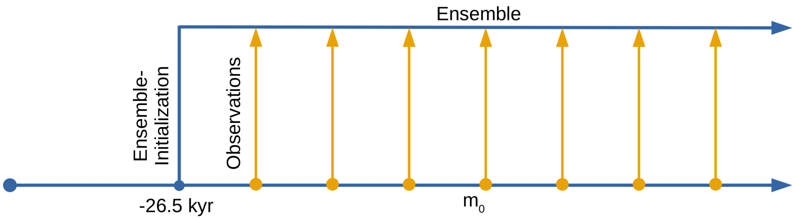


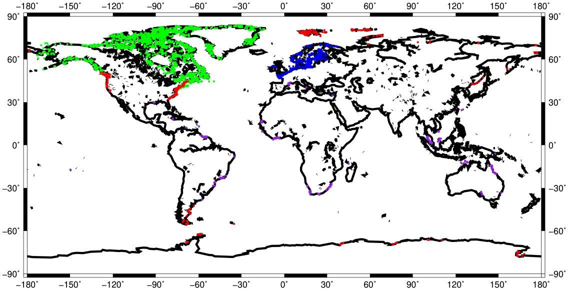
Identical twins

- Reference run m_0 with target viscosity values
- Ensemble initialization from reference model at 26.5 / 10.5 kyrs BP
- Observations at regular time intervals (1 kyr)
- Synthetic observations at locations where real observations exist



Observations

Locations of real observations, projected onto VILMA grid points:



Region	Num. of observations
Global	1807
NA & Greenland	1309
Fennoscandia	209

Results Part I: The 3-layer model

Investigate dependence on:

- Observation uncertainty
- Observation distribution
- Observation period

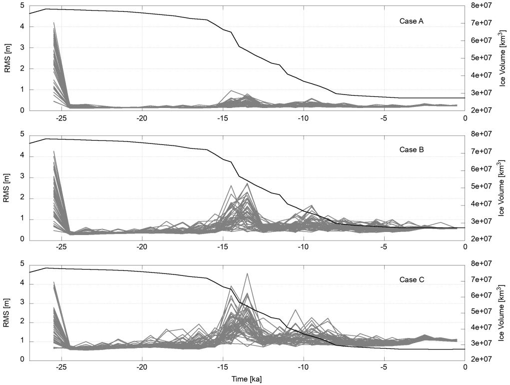
Observation uncertainty

Obs. uncert.

0.1 m

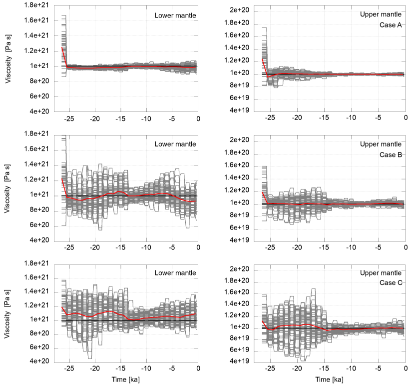
0.25 m

0.5 m



RMSE development for RSL (grey)

Ice volume (black)



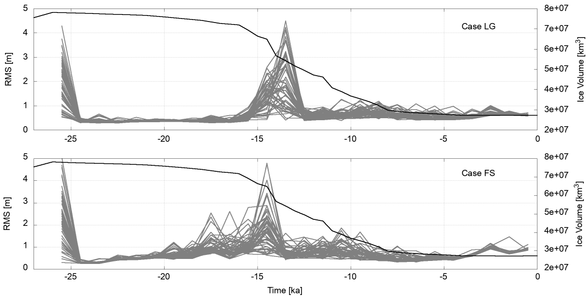
Viscosity development (grey)

Ensemble mean (red)

Target values (black)

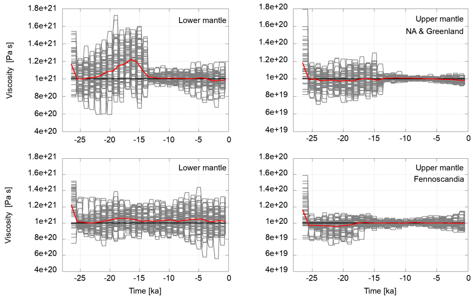
Observation distribution

Obs. uncertainty: 0.25 m (same as case B)



RMSE development for RSL (grey)

Ice volume (black)



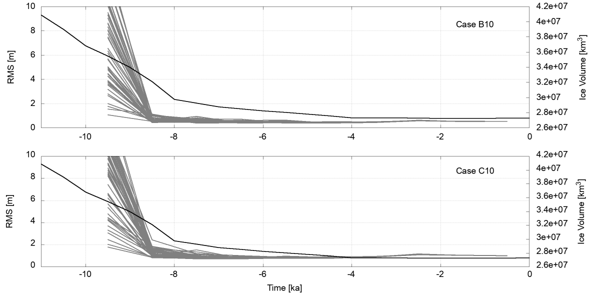
Viscosity development (grey)

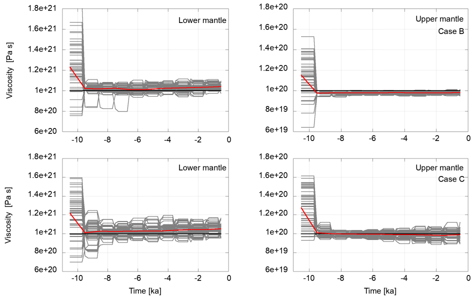
Ensemble mean (red)

Target values (black)

10 kyrs of observations

Global data set, observations from 10 ka BP till present day





RMSE development for RSL (grey)
Ice volume (black)

Obs. uncertainty B10: 0.25 m
C10: 0.5 m

Viscosity development (grey)
Ensemble mean (red)
Target values (black)

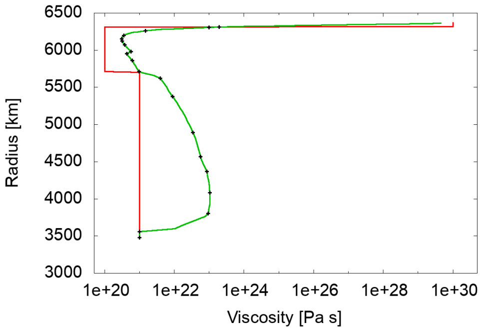
Results Part II: The 1D-profile model

Perturbation strategies:

1. Scaling entire profile with common factor
2. Adjusting individual layers
3. Combination of 1 & 2

3-layer model vs. 1D profile

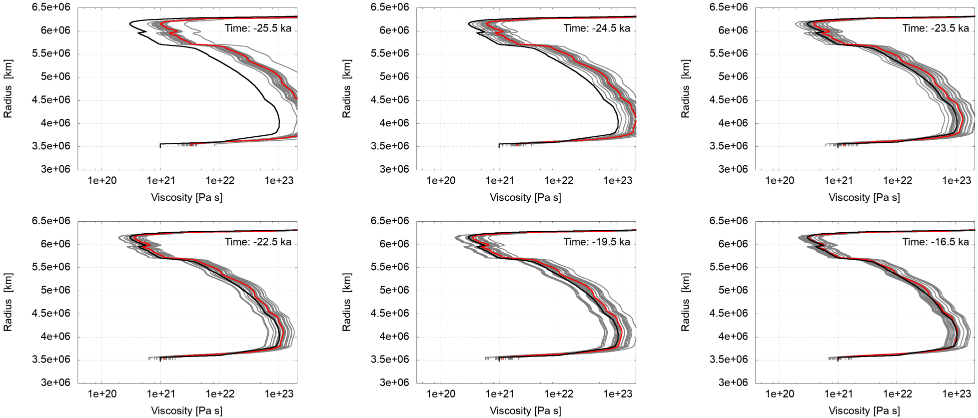
Comparison 3-layer model (red) vs. 1D profile (green)



1D profile:

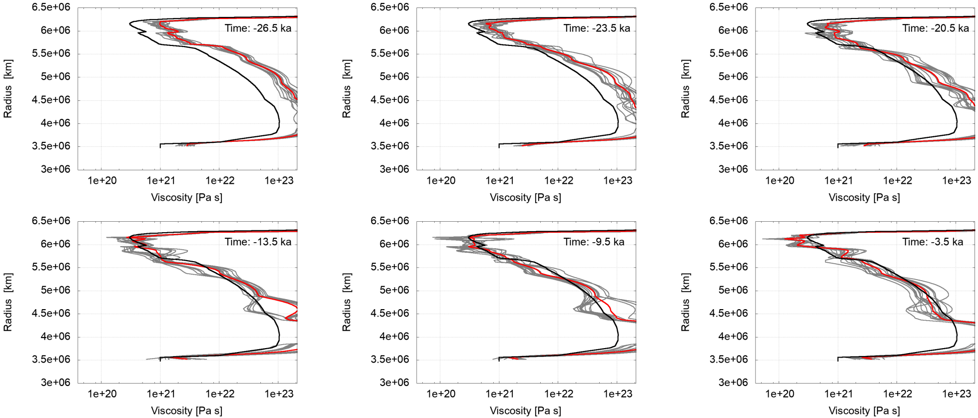
- 12 fixed lithospheric layers
- 152 viscous mantle layers
- Viscosity in mantle layers parameterized with cubic hermite splines to ensure smoothness (20 knots)
- Perturbation of viscosity values of spline knots (black crosses) during assimilation
- Values for layers obtained by spline interpolation

1D profile: scaling



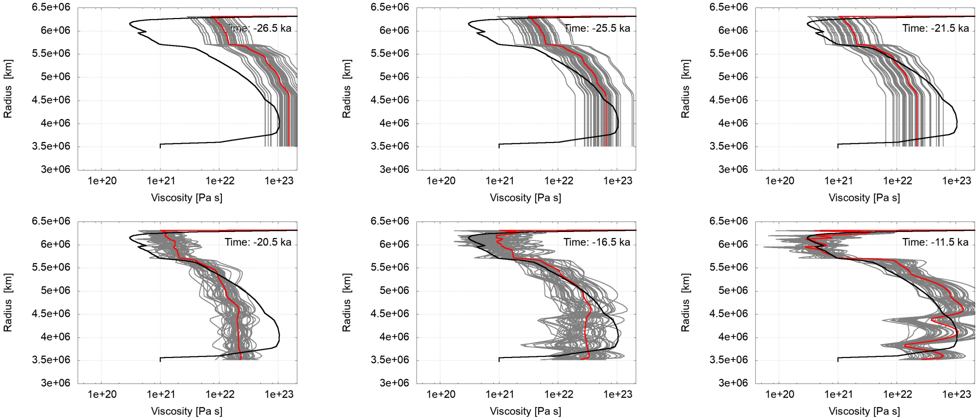
Black: target profile, grey: ensemble models, red: ensemble mean

1D profile: spline parameterization



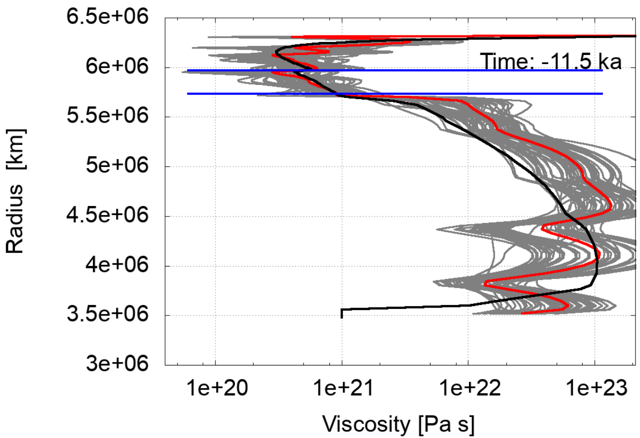
Black: target profile, grey: ensemble models, red: ensemble mean

1D profile: scaling + profile shape adjustment



Black: target profile, grey: ensemble models, red: ensemble mean

Known discontinuities



Summary

- Sucessfully applied particle filter for mantle viscosity estimation in a sandbox setup
- Very good convergence to target values in the 3-layer parameterization
- For depth profile good convergence for shallow layers, deep layer viscosities are more difficult to constrain $\rightarrow$ slower convergence

Next steps

- Improve profile smoothness, i.e. adapt perturbation scheme
- Steps towards a more realistic temporal observation distribution
- Account for temporal observation uncertainties
- Couple to ice model for joint assimilation

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Selected references

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