Ensemble Methods: Challenges Faced In and Lessons Learned From Practical Applications

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Outline

- History matching in oil and gas
- Implementation of ESMDA
- Probabilistic history matching example
- Deterministic history matching example
- Quantifying modeling error

History matching problem in the oil and gas industry

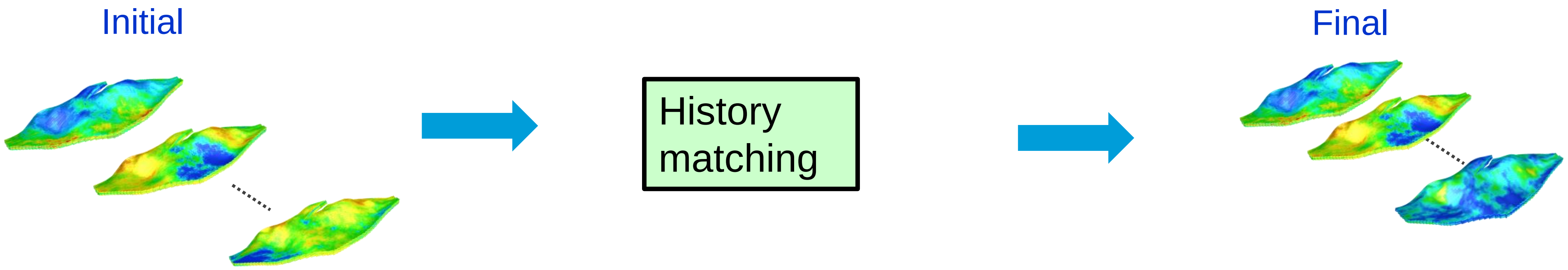
- Governing equations for subsurface oil and gas flow (mass conservation)

$$\frac{\partial}{\partial t}\left(\phi\rho_j S_j\right) - \nabla\cdot\left[\rho_j\lambda_j\mathbf{k}\left(\nabla p_j - \rho_j g\nabla D\right)\right] + q_j^w = 0$$

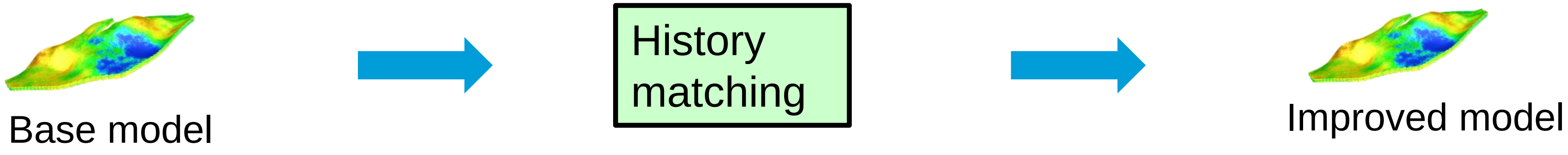
- Reservoir simulations are driven by uncertainties, not chaotic behaviors
  - State: Pressure (p), saturation (S), …
  - Observation ($\boldsymbol{d}$): Production/injection data at the wells (q), …
  - Uncertainties ($\boldsymbol{m}$): Porosity (ϕ) and permeability (k) distribution, fluid mobility (λ), compressibility, geological structure, facies, fluid PVT … Can be continuous or categorical
- History matching (a.k.a., data assimilation in petroleum engineering):
  - Calibrate the uncertainty parameters ($\boldsymbol{m}$, usually non-Gaussian) to production history ($\boldsymbol{d}$, usually nonlinear to $\boldsymbol{m}$) to reduce uncertainty and improve accuracy in the forecast.

Probabilistic History Matching vs Deterministic History Matching

- Probabilistic history matching: Prior distribution in, posterior distribution out
  - Popular methods: Design of experiment + proxy + Monte Carlo, ensemble methods
  - Does ensemble-based method provide reliable estimate of posterior uncertainty?



- Deterministic history matching: One model in, one model out
  - Popular methods: Optimization-based algorithms such as the genetic algorithms and adjoint
  - Can ensemble-based method be used for deterministic history matching?



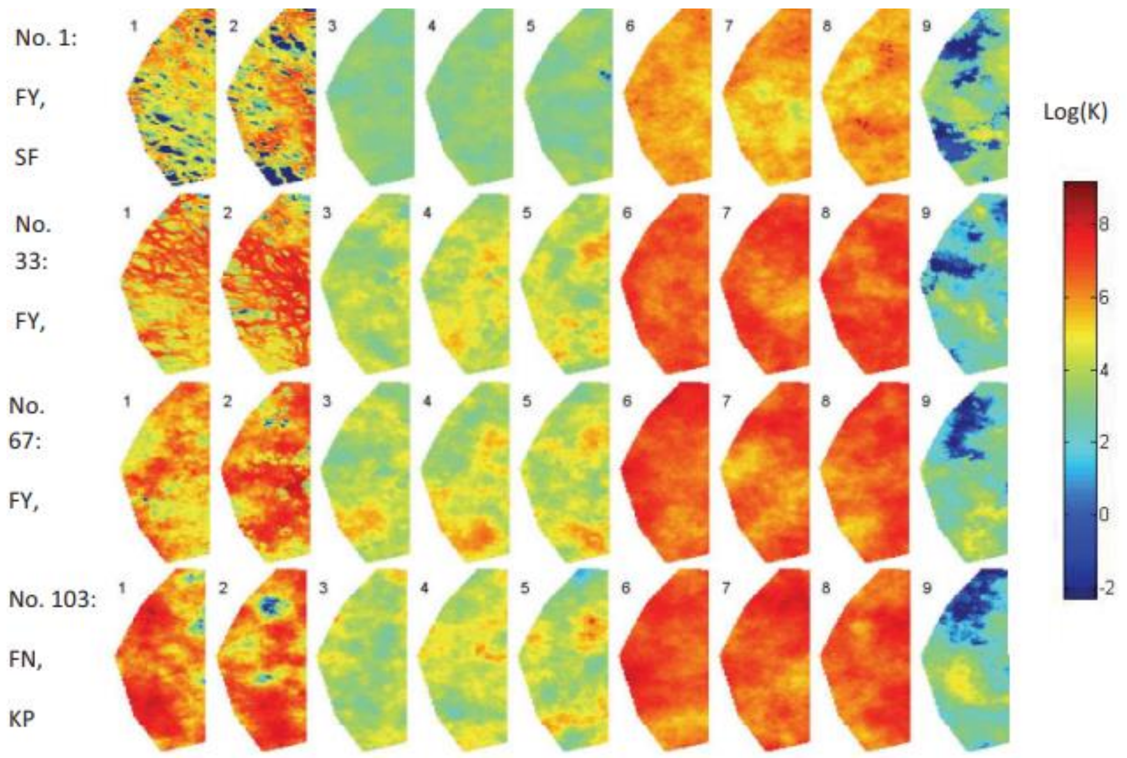
Parameter-Based vs Realization-Based Characterization

- Characterization of Uncertainties: Parameter-based vs Realization-based

Parameter-based*

	Variables	Description	Minimum	Maximum
1	SORW	OW rel. perm. end point	0.1	0.2
2	KRWRO	OW rel. perm. end point	0.6	0.9
3	KROCW	OW rel. perm. end point	0.8	1
4	WEXP	OW rel. perm. exponent	1	4
5	OWEXP	OW rel. perm. exponent	1	4
6	RCOMP	Rock compressibility	3E-06	4E-06
7	WOC	Oil-water contact	5575	5580
8	PERM1	Layers 1–5 perm. multiplier	0.5	5
9	PERM2	Layers 6–9 perm. multiplier	0.5	5
10	PORO1	Layers 1–5 porosity multiplier	0.6	1.5
11	PORO2	Layers 6–9 porosity multiplier	0.85	1.15

Realization-based characterization**



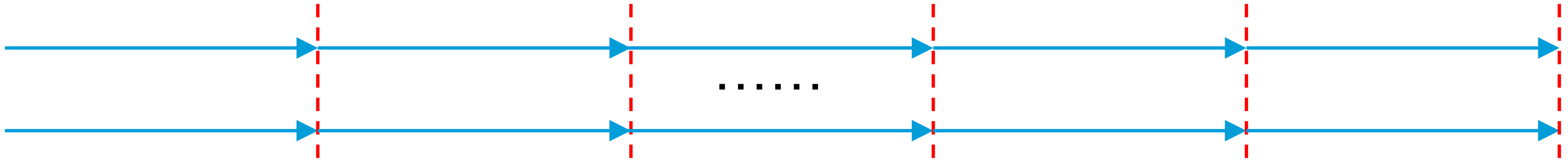
- Challenge:
  – Not all parameters are spatial
  – How to incorporate modeling errors in parameterization

*He, Jincong, et al. "Quantifying expected uncertainty reduction and value of information using ensemble-variance analysis." SPE Journal 23.02 (2018): 428-448

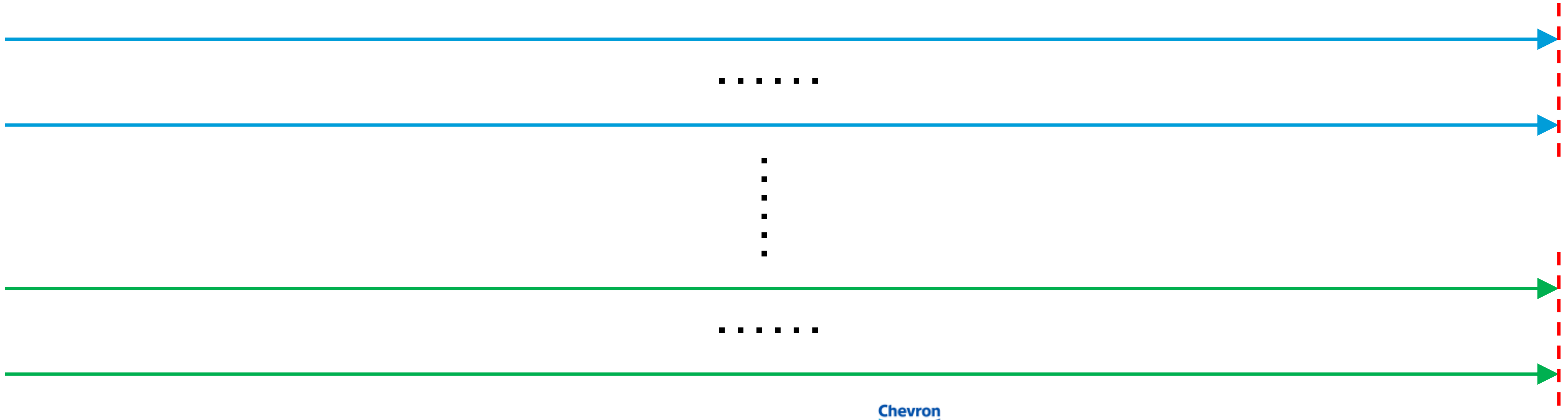
**Peters, Lies, et al. "Results of the Brugge benchmark study for flooding optimization and history matching." *SPE Reservoir Evaluation & Engineering* 13.03 (2010): 391-405.

Comparison to Other Ensemble-Based Methods

- Ensemble Kalman filter (EnKF)
  - Cons: Difficult to implement; Unphysical update (e.g., negative saturation, see Dr. Pfander's talk); Inconsistent with governing equations



- Iterative ensemble smoother
  - Pros: No need to update states; Use simulator as black box; Consistent with the governing equations



Ensemble Smoother with Multiple Data Assimilation

- ESMDA formula for uncertain parameter update (Emerick and Reynolds 2013)*

$$\boldsymbol{m}_i^{n+1} = \boldsymbol{m}_i^n + \boldsymbol{C}_{md}^n(\boldsymbol{C}_{dd}^n + \alpha_n \boldsymbol{C}_e^n)^{-1}[\boldsymbol{d}_i^n - (\boldsymbol{d}_{obs}^n + \boldsymbol{e}_i^n)]$$

Parameters after update ($\boldsymbol{m}_i^{n+1}$)

Parameters before update ($\boldsymbol{m}_i^n$)

Kalman gain ($\boldsymbol{C}_{md}^n(\boldsymbol{C}_{dd}^n + \alpha_n \boldsymbol{C}_e^n)^{-1}$)

Mismatch ($[\boldsymbol{d}_i^n - (\boldsymbol{d}_{obs}^n + \boldsymbol{e}_i^n)]$)

*Emerick, Alexandre A., and Albert C. Reynolds. "Ensemble smoother with multiple data assimilation." *Computers & Geosciences* 55 (2013): 3-15.

- α controls the step size of the update: It satisfies $\sum \frac{1}{\alpha_n} = 1$
  - Taking a journey in multiple steps to tackle nonlinearity
- It is proven that the final ensemble follows posterior uncertainty when
  - Linear-Gaussian problem with an infinite-sized ensemble
- Possible alternatives: D-ESMDA (Emerick 2018), EnRML (Raanes 2019, Chen and Oliver 2012)

Localization

- Kalman gain could suffer from spurious correlations that lead to ensemble collapse
- Factors that seem to aggravate the ensemble collapse
  - (1) Small ensemble size, (2) Large number of (redundant) data point, (3) Small degree of freedom
  - (4) Error/tolerance too small, (5) Simulation response inconsistent with the data
- Practical requirement for the location scheme
  - Simple: Minimal user input, minimal information needed
  - Robust: Able to handle different type of uncertainties/data; Adaptive and self-learning
- Recent work
  - Correlation-based adaptive localization (Zhang and Oliver, 2010; Anderson, 2016; Luo et al. 2018)
  - A comprehensive review and comparison (Chen and Oliver, 2016)

Bootstrap-Based Localization

- Ideas: Dampen inconsistent elements in the Kalman as identified by bootstrap sampling
- Procedures
  - Original Kalman gain: $\boldsymbol{K} = \boldsymbol{C}_{md}^{n}(\boldsymbol{C}_{dd}^{n} + \alpha_n \boldsymbol{C}_{e}^{n})^{-1}$
  - Step 1. Perform bootstrap sampling on the original ensemble to create n_b bootstrapped ensembles
  - Step 2. For each bootstrapped ensemble calculate the Kalman gain matrix
  - Step 3. Calculate the mean and variance of each Kalman gain elements across all bootstrapped ensembles
  - Step 4. Dampen elements of the original Kalman gain according to level of inconsistency

$$\boldsymbol{K}_{ij}^{s} = \frac{1}{1 + 4\boldsymbol{\sigma}_{K_{ij}}^{2} / \overline{\boldsymbol{K}}_{ij}^{2}} \boldsymbol{K}_{ij}$$

*Zhang, Y. and Oliver, D. (2010). Improving the ensemble estimate of the Kalman gain by bootstrap sampling, Math Geosci, 42, 327-345.

Chevron

Efficient Calculation of Kalman Gain

- Bootstrap-based localization requires repeated calculating of Kalman gain

$$\boldsymbol{K} = \boldsymbol{C}_{md}^{n}(\boldsymbol{C}_{dd}^{n} + \alpha_n \boldsymbol{C}_{e}^{n})^{-1}$$

- When the number of data (n_d) is smaller (e.g., $n_d < 500$)
  - If n_d is the smallest among n_d, n_m and n_r
    - Calculate the inverse directly
  - If n_m is the smallest among n_d, n_m and n_r
    - Solve n_m linear equations $(\boldsymbol{C}_{dd}^{n} + \alpha_n \boldsymbol{C}_{e}^{n})\boldsymbol{K} = \boldsymbol{C}_{md}^{n}$
  - If n_r is the smallest among n_d, n_m and n_r
    - Solve n_r linear equations $(\boldsymbol{C}_{dd}^{n} + \alpha_n \boldsymbol{C}_{e}^{n})\boldsymbol{X} = \boldsymbol{D}$, then $\boldsymbol{K} = \frac{\boldsymbol{1}}{\boldsymbol{n_r - 1}}\boldsymbol{MX}$
- When the number of data (n_d) is large
  - Use the subspace method*

*Emerick, A.A., 2016. Analysis of the performance of ensemble-based assimilation of production and seismic data. *Journal of Petroleum Science and Engineering*, *139*, pp.219-239.

Subspace Method

- Need to efficiently evaluate Kalman gain

$$\boldsymbol{K} = \boldsymbol{C}_{MD}^{n}(\boldsymbol{C}_{DD}^{n} + \alpha_n \boldsymbol{C}_{e}^{n})^{-1}$$

- Procedure of the subspace method*
  - Scale the data with error (matrix $\boldsymbol{S}$)
  - Perform SVD on data, only keep significant components ($\boldsymbol{D} = \boldsymbol{U}_r \boldsymbol{W}_r \boldsymbol{V}_r^T$)
  - Perform SVD on the error term to diagonalize it (matrix $\boldsymbol{H}_r$)
  - Use pseudo inverse instead of actual inverse

$$(\boldsymbol{C}_{DD}^{n} + \alpha_n \boldsymbol{C}_{e}^{n})^{-1} \approx (N_e - 1)\boldsymbol{S}^{-1}\boldsymbol{U}_r \boldsymbol{W}_r^{-1}\boldsymbol{Z}_r[\boldsymbol{I}_r + \boldsymbol{H}_r]^{-1}\left(\boldsymbol{S}^{-1}\boldsymbol{U}_r \boldsymbol{W}_r^{-1}\boldsymbol{Z}_r\right)^{\boldsymbol{T}}$$

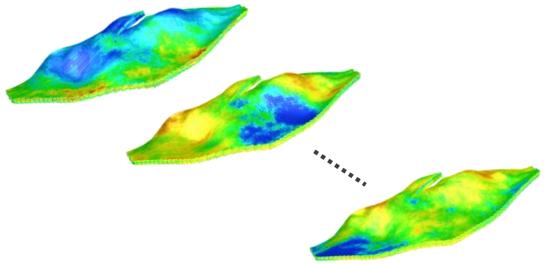
- Not necessarily good approximation of the original problem (See Dr. Raanes's talk)
- It is more robust and an enabling method for the bootstrap-based localization

*Emerick, A.A., 2016. Analysis of the performance of ensemble-based assimilation of production and seismic data. *Journal of Petroleum Science and Engineering*, *139*, pp.219-239.

ESMDA for Probabilistic History Matching

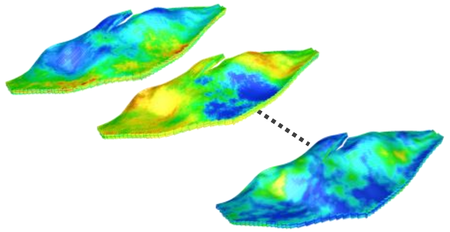
- ESMDA is proven to sample the posterior correctly in idealized scenarios
- How would the method perform for probabilistic history matching for semi-realistic cases

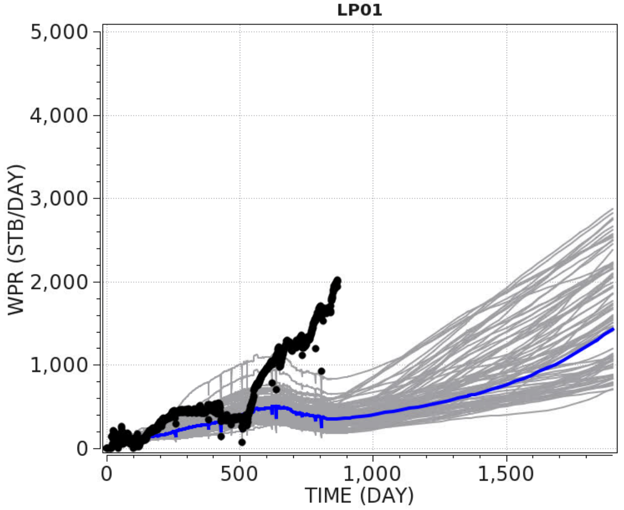
Initial



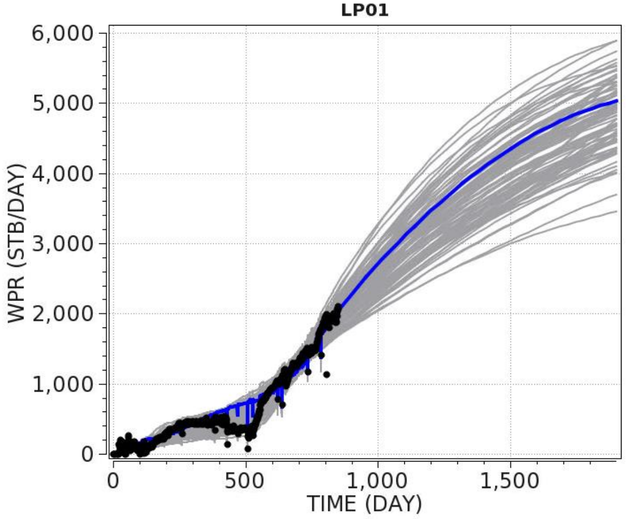
ESMDA

Final





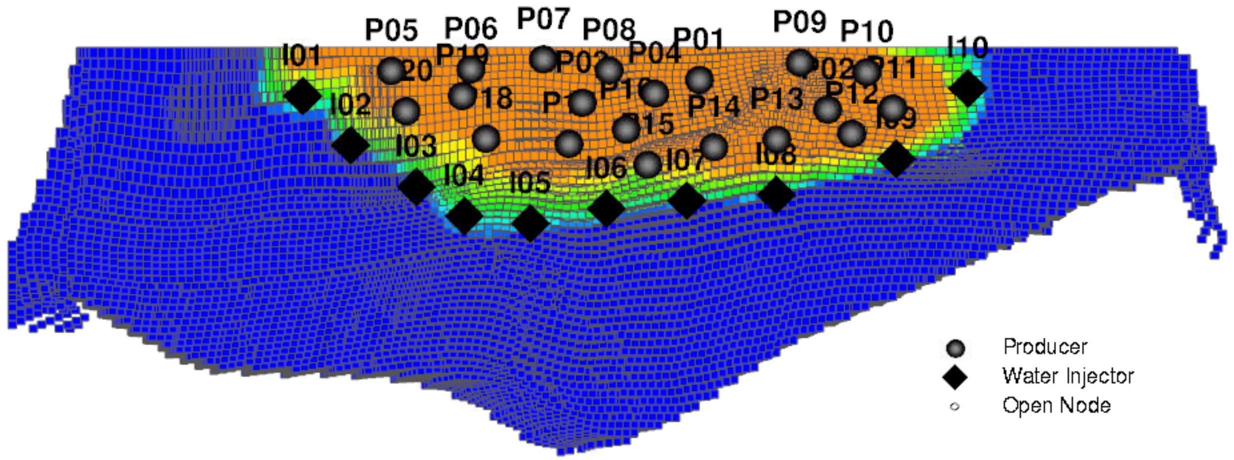
Before history matching



After history matching

Example 1. Synthetic Problem with the Brugge Reservoir Model*

- Uncertainty parameters (92 parameters)
  - Reservoir divided into 30 regions
    - Horizontal permeability multiplier (k_h)
    - Vertical permeability multiplier (k_v)
    - Pore volume multipliers (PV)
  - Two dummy variables
    - Independent of other uncertainties, have no impact on the simulation results
    - Use to detect ensemble collapse and evaluate uncertainty analysis quality
- Data to assimilate
  - 10 years of production from 30 wells
  - Well oil production rate (OPR) and bottom-hole pressure (BHP), (700+ points)
  - 100-sized ensemble and 4 iterations are used in ESMDA

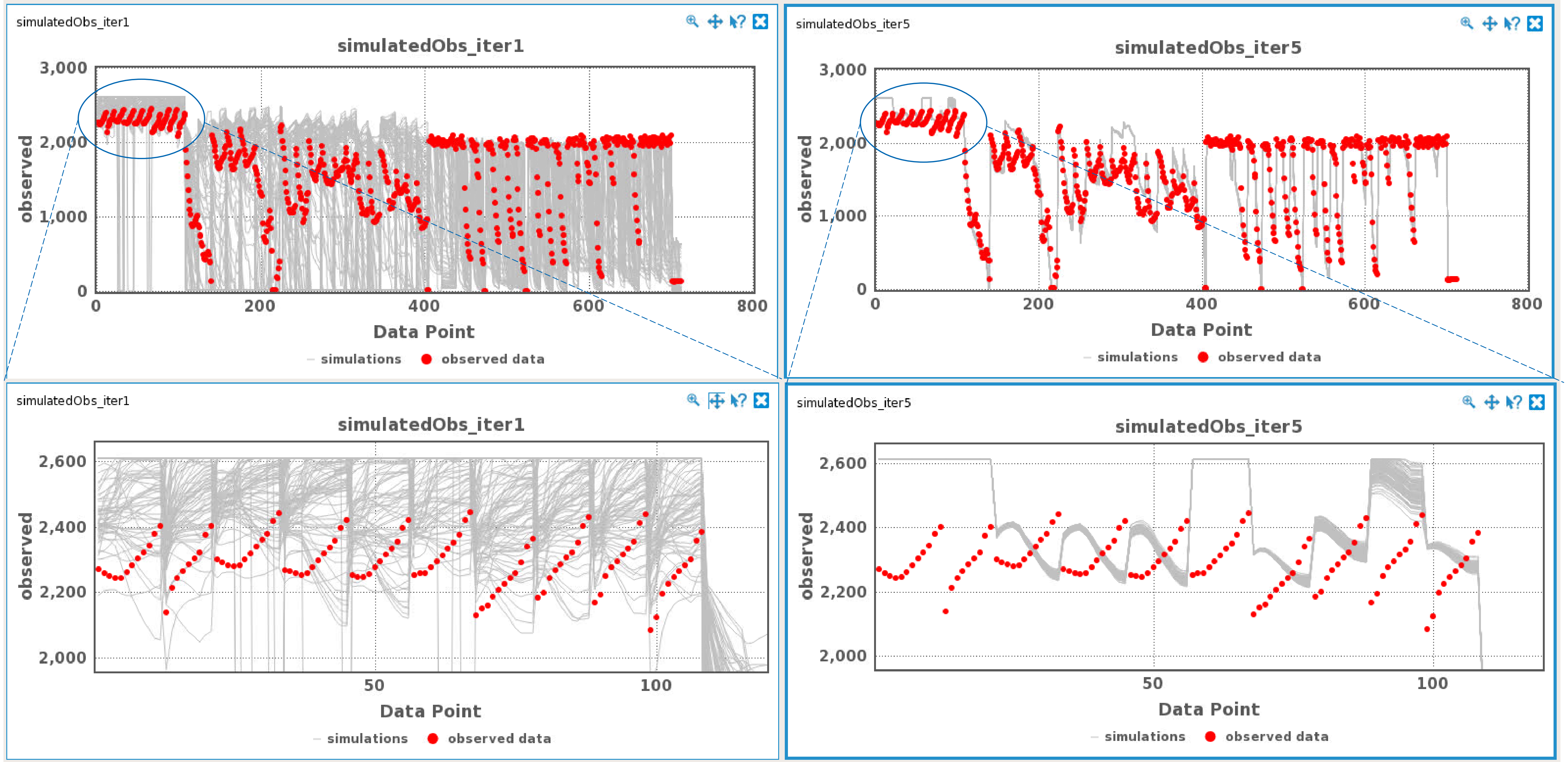


Synthetic Brugge waterflood problem

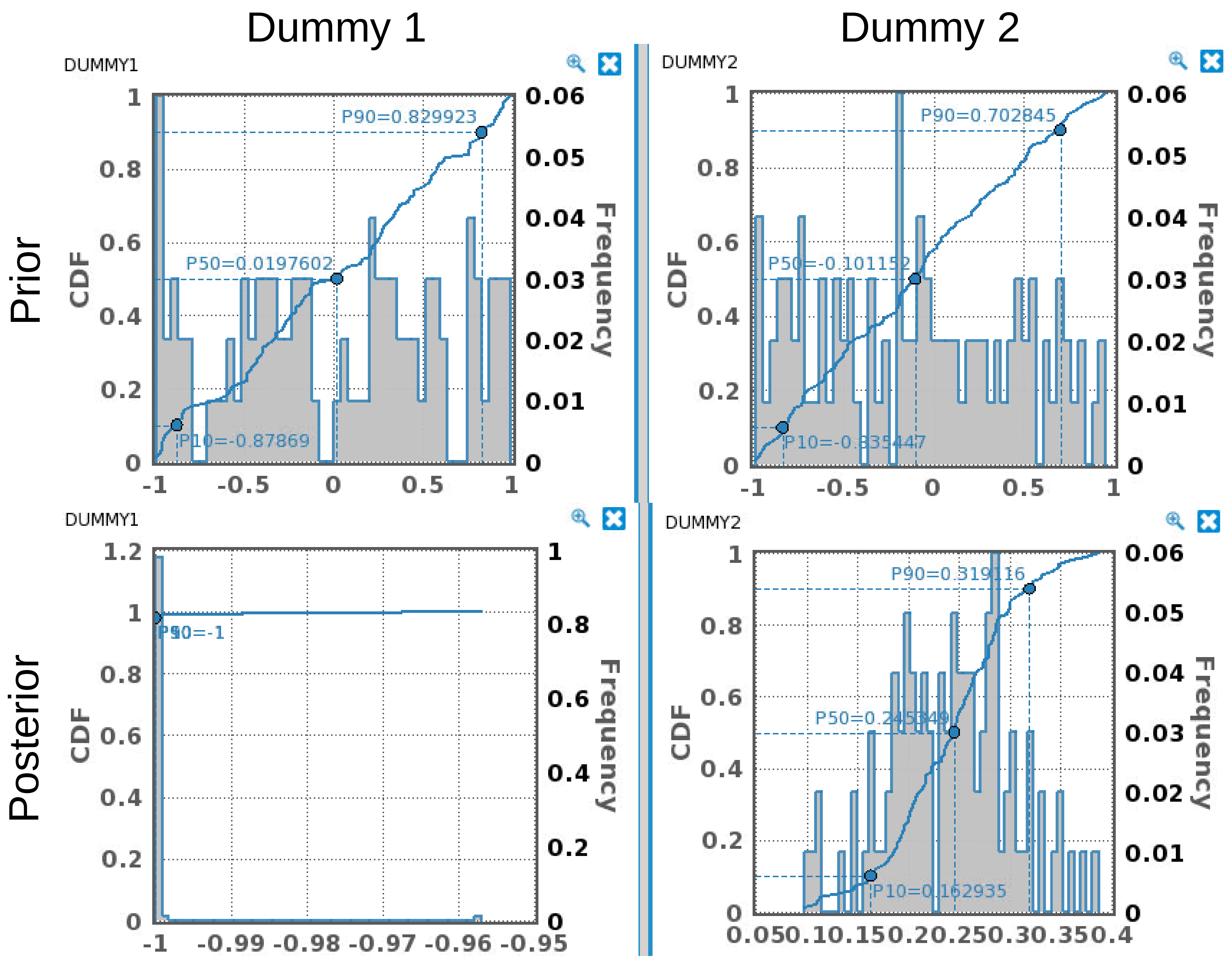
*Peters, Lies, et al. "Results of the Brugge benchmark study for flooding optimization and history matching." *SPE Reservoir Evaluation & Engineering* 13.03 (2010): 391-405.

Results with $N_e = 100$ and No Localization

- All data from all wells, concatenated together



Change in Dummy Parameters



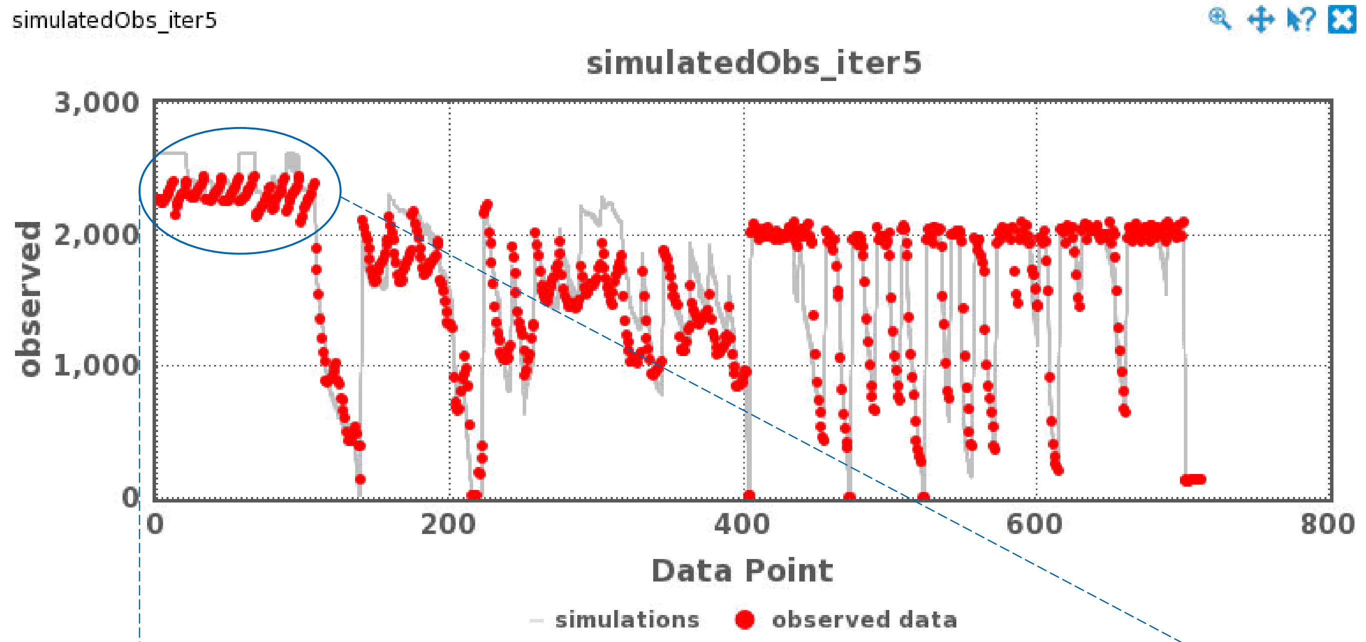
- Change in dummy parameters shows collapse of the ensemble
- Just looking at one dummy parameter is not enough

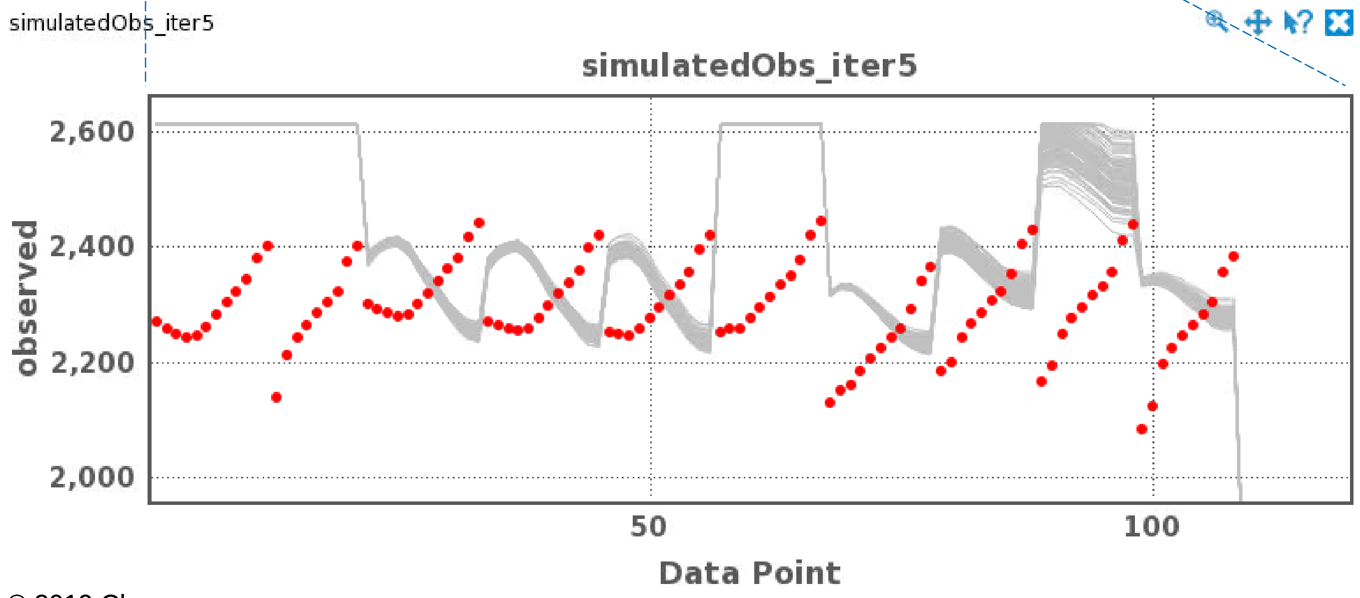
	Dummy 1	Dummy 2
Mean Shift	-1.02	0.14
Uncertainty reduction	100%	90.3%

Localization Helps

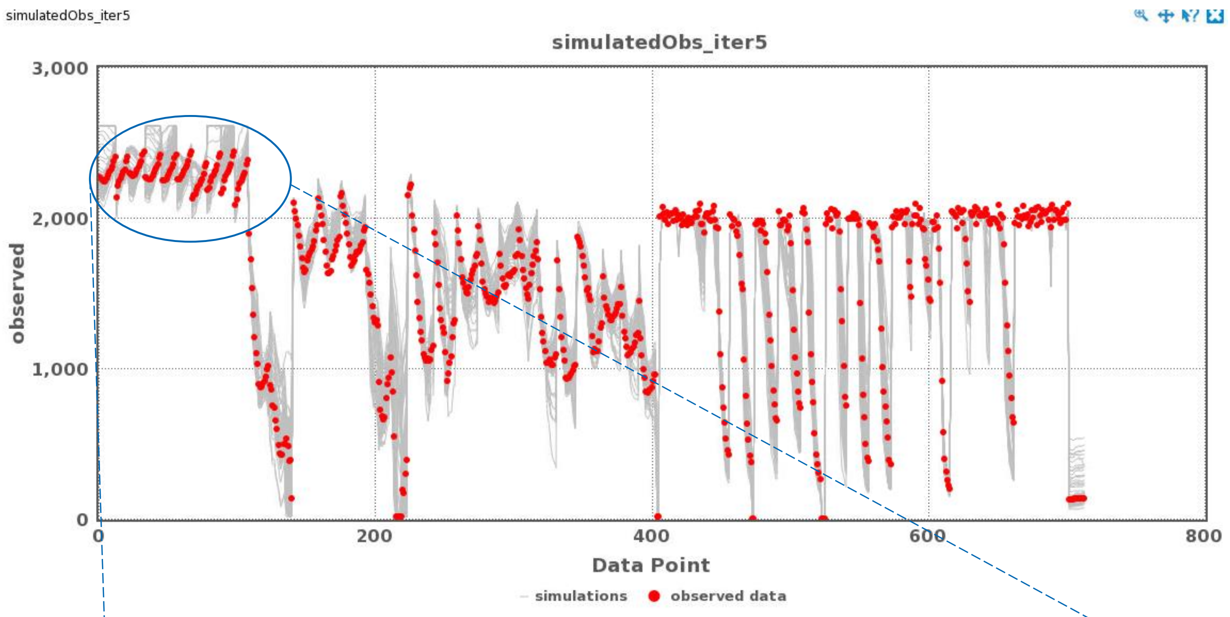
- All data from all wells, concatenated together

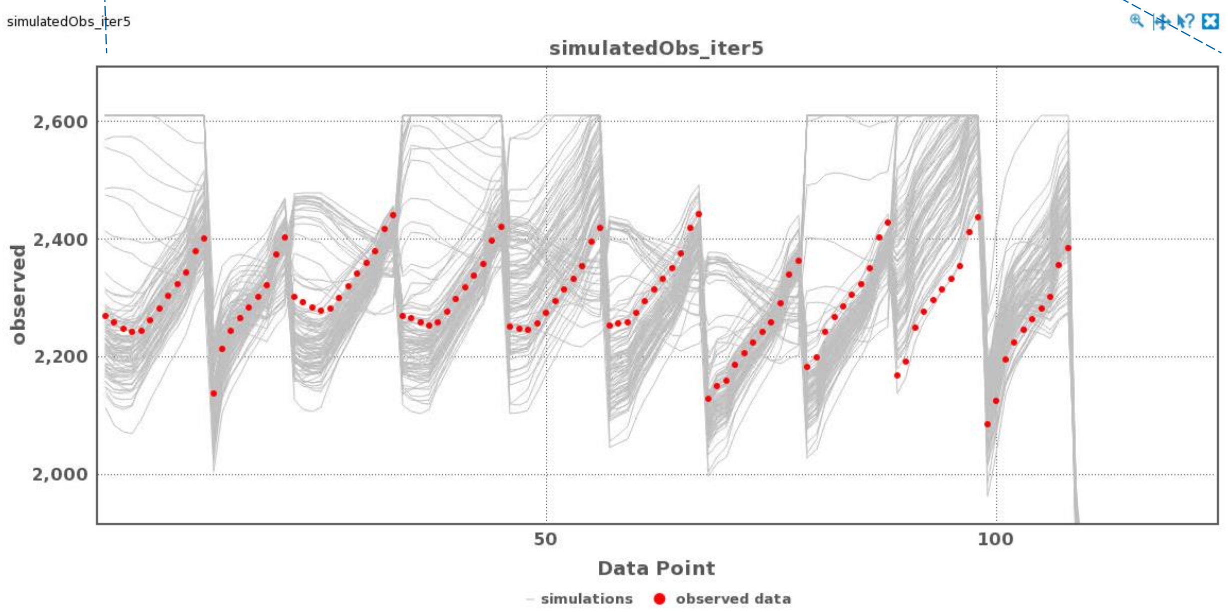
w/o localization



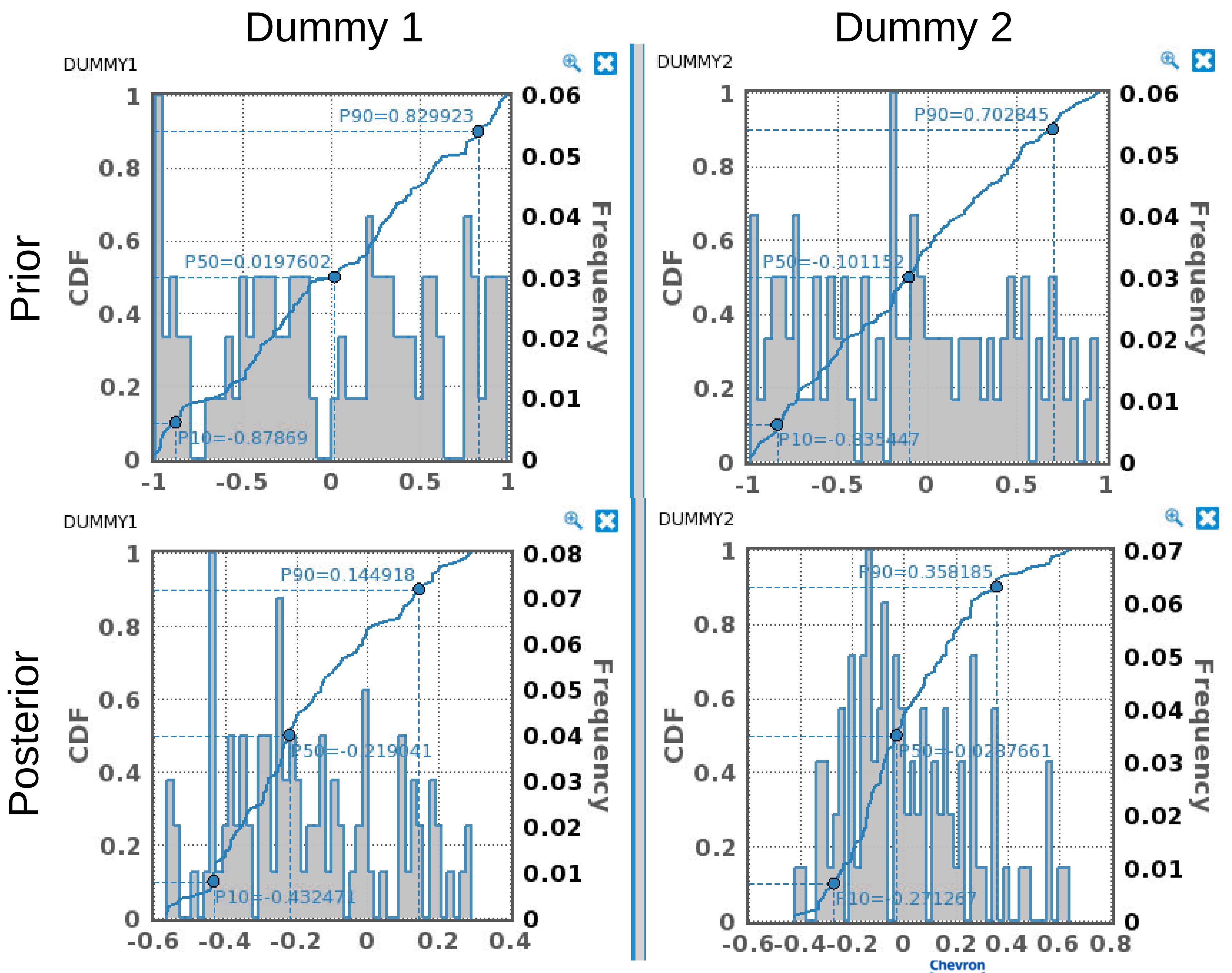


w/ localization





Change in Dummy Parameters w/ Localization

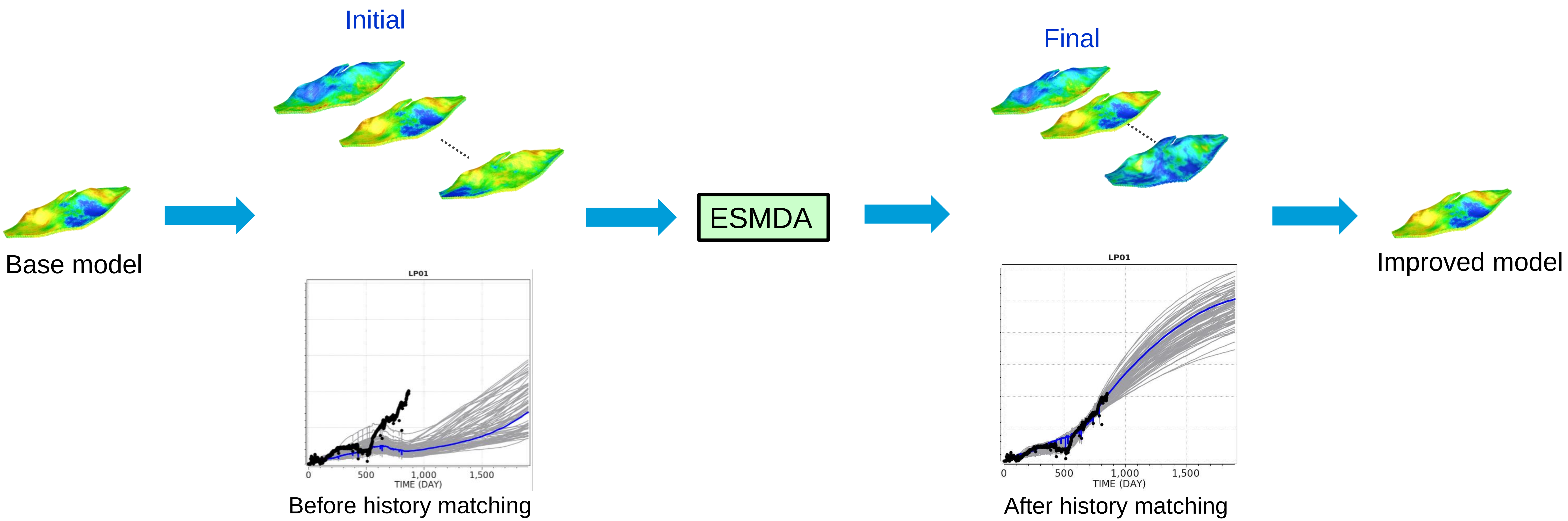


- Ensemble collapse is alleviated but not fully eliminated
- Could be further improved with larger ensemble
- Current implementation not reliable for probabilistic forecast
- See talks by Dr. Aanonsen and Dr. Bjarkason

	Dummy 1	Dummy 2
Mean Shift	-0.2	0.08
Uncertainty reduction	66%	40.9%

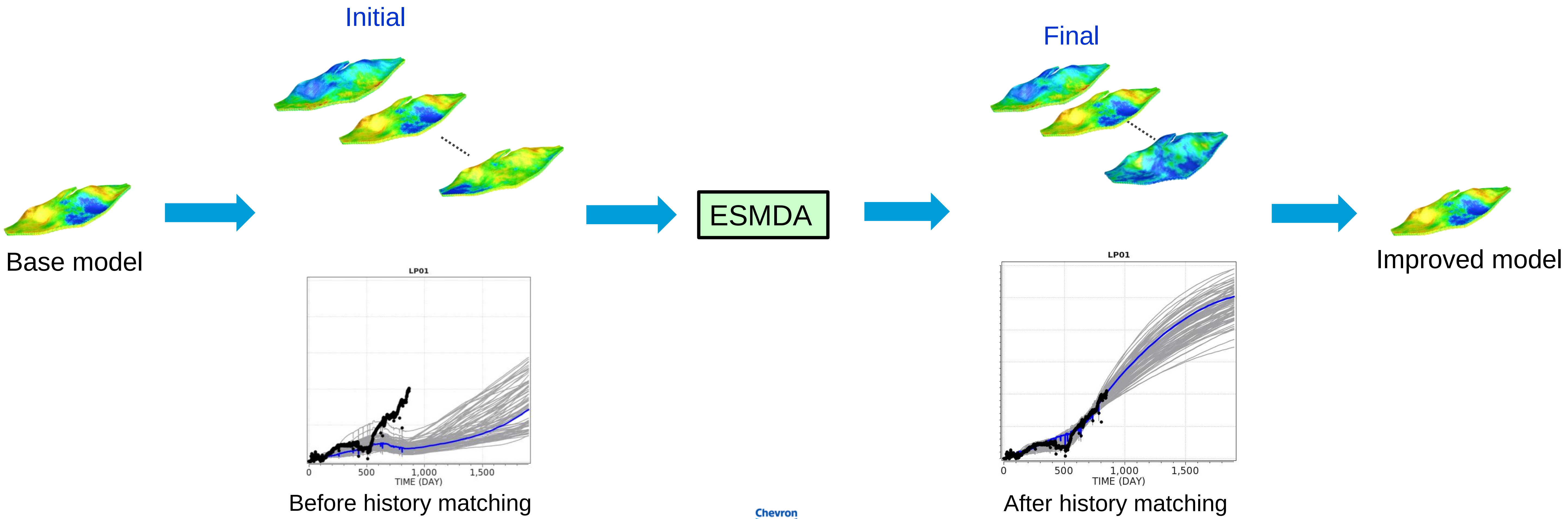
ESMDA for Deterministic History Matching

- Input: A base model with proper parameterization of uncertainties
- Output: An improved model that better matches the history with minimal update
- Challenge: How to select the “improved model” from the final iteration



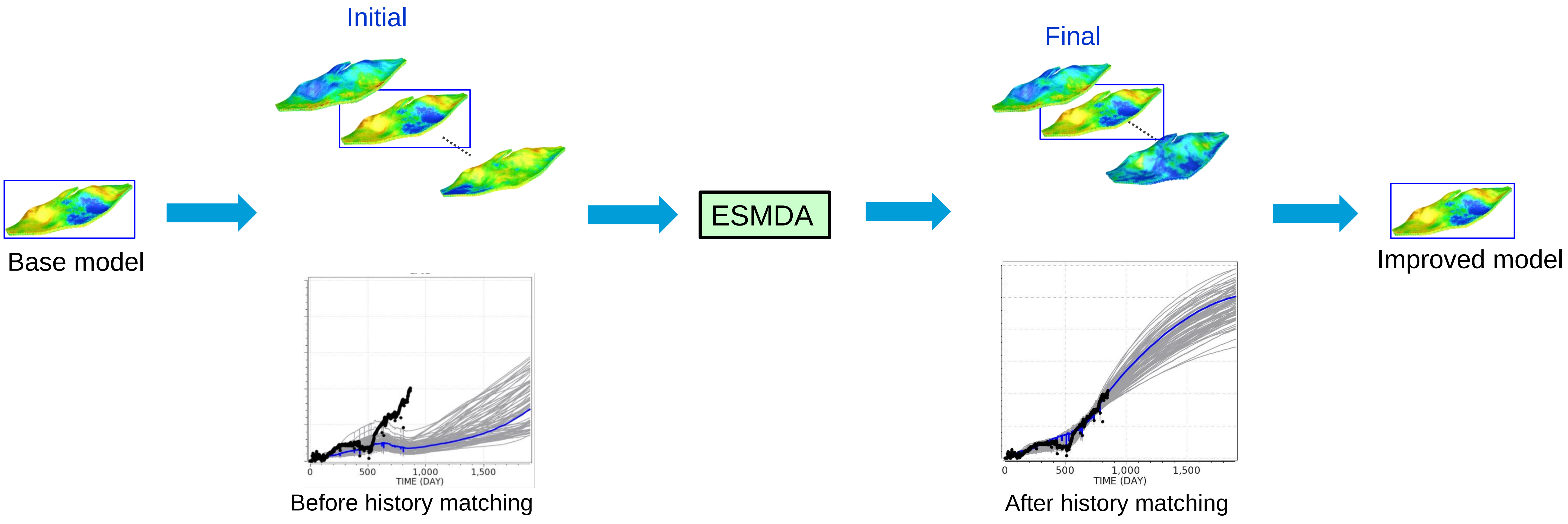
ESMDA for Deterministic History Matching

- Challenge: How to select the “improved model” from the final iteration
  - Option 1. Choose the one with the best match…Could be overfitting
  - Option 2. Take the mean of the ensemble…Violate statistics
  - Option 3. Choose base on P50 of the prediction…Still could be overfitting



ESMDA for Deterministic History Matching (Model Maturation)

- Challenge: How to select the “improved model” from the final iteration
  - Option 4. Include the base model in the prior ensemble, choose the corresponding posterior model

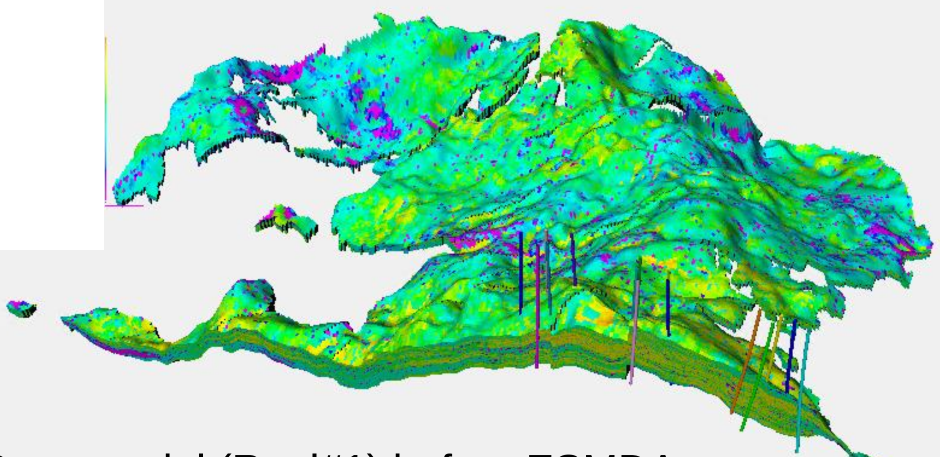


Example 2: ESMDA for Deterministic History Matching

- General model maturation workflow

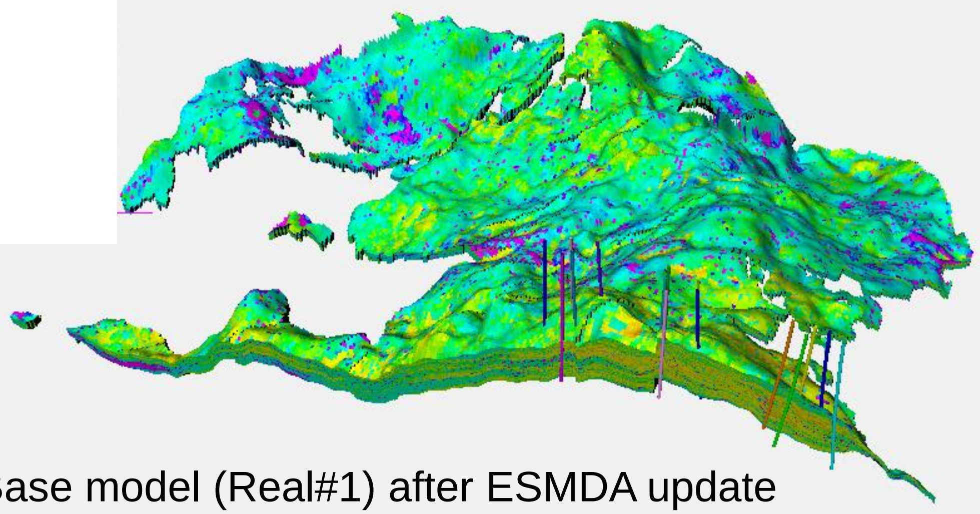
Base model → Perturb uncertainties to generate a prior ensemble → ESMDA → Select updated model from posterior ensemble

- History matching the permeability($\boldsymbol{k}$) and porosity ($\boldsymbol{\phi}$) field in Reservoir X
  - 8 million active cells. 16 million uncertainty parameters in total
  - Ensemble generated by perturbation upon the base model through sequential Gaussian simulation
  - Data to match: Well BHP at one of the producers



Base model (Real#1) before ESMDA

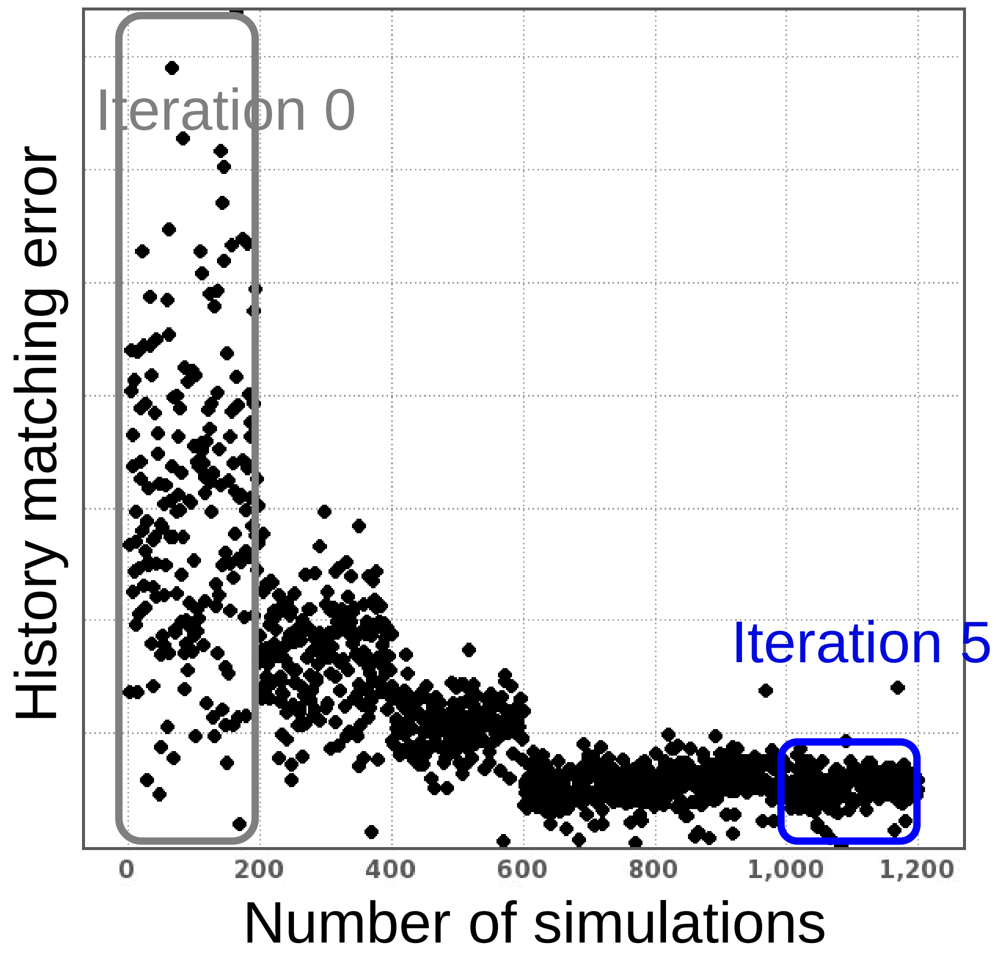
→ ESMDA →

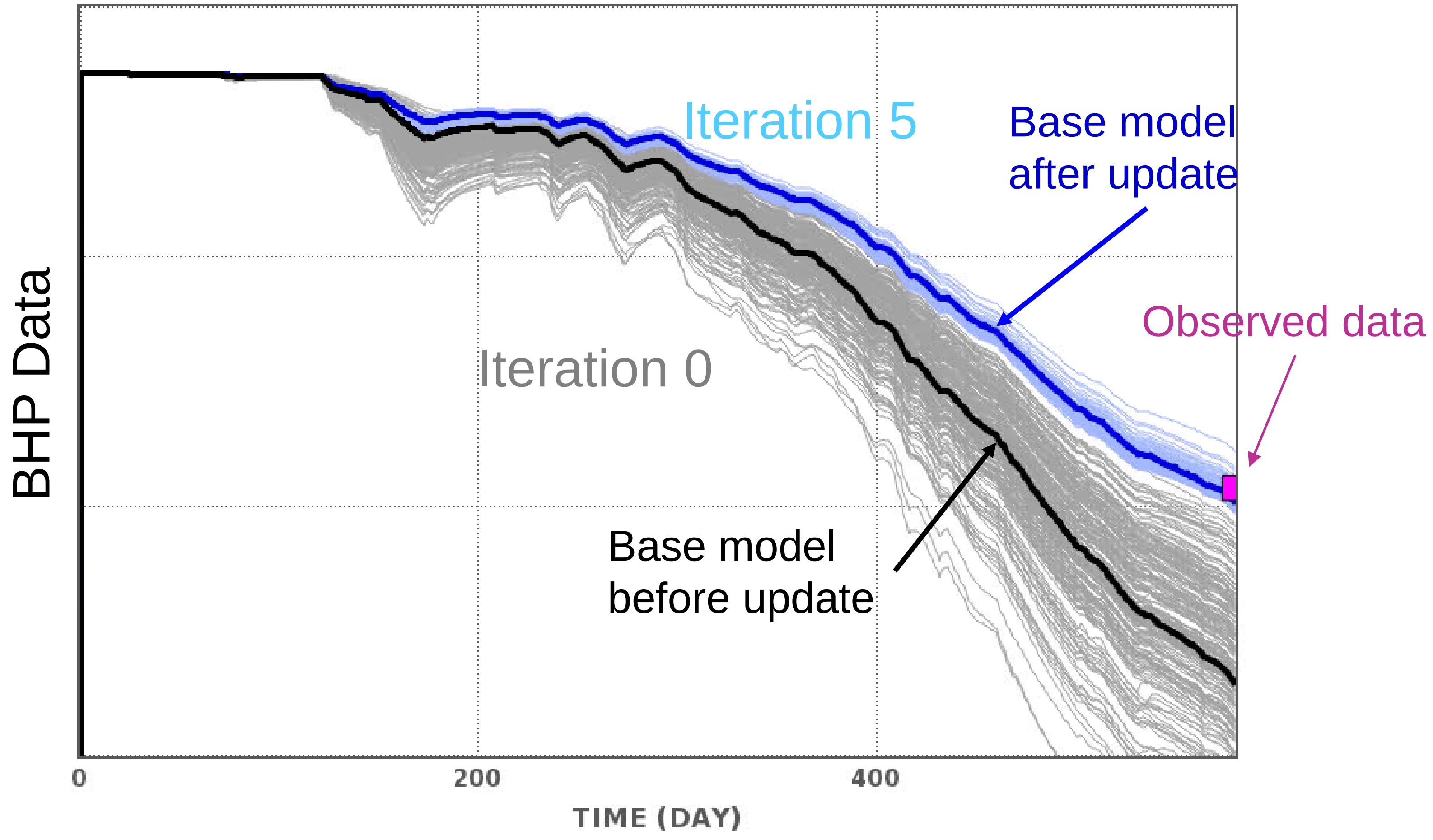


Base model (Real#1) after ESMDA update

History Matching Quality

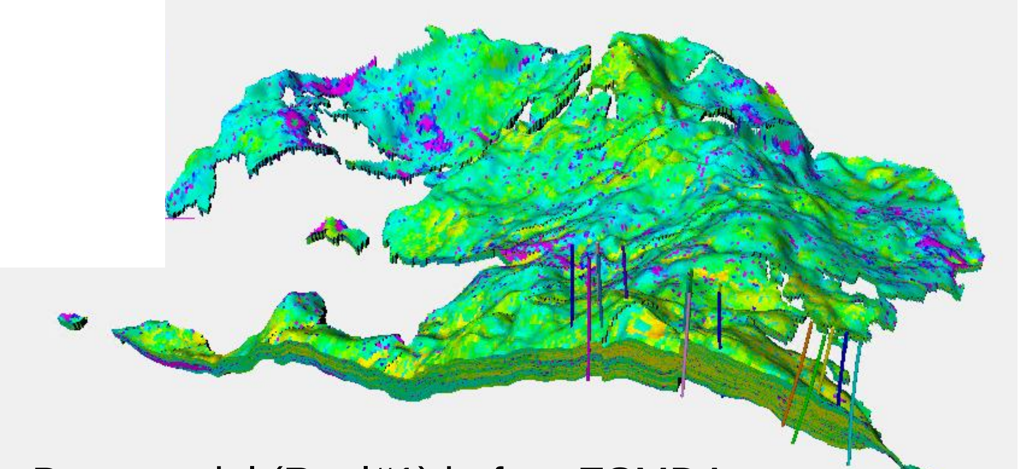
- 200-sized ensemble and 5 iterations are used
- A total of 1200 simulations are run



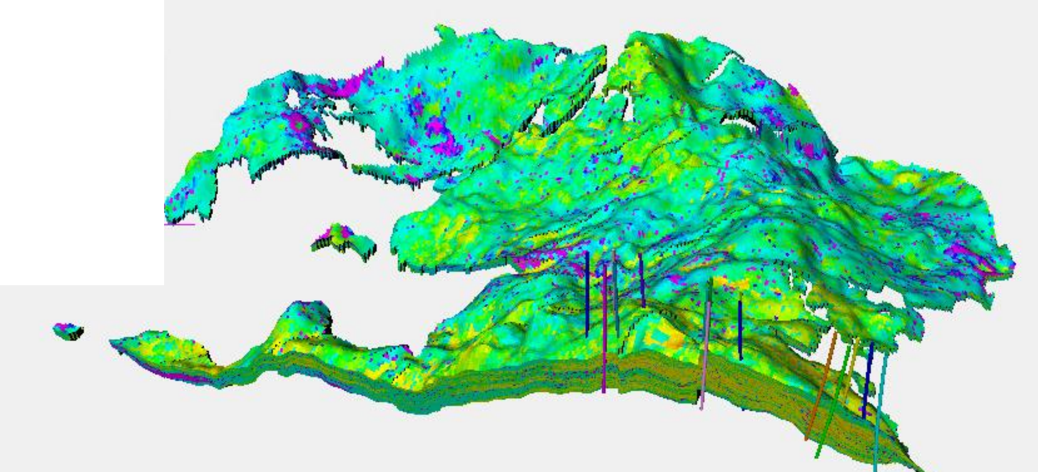


ESMDA for Deterministic History Matching

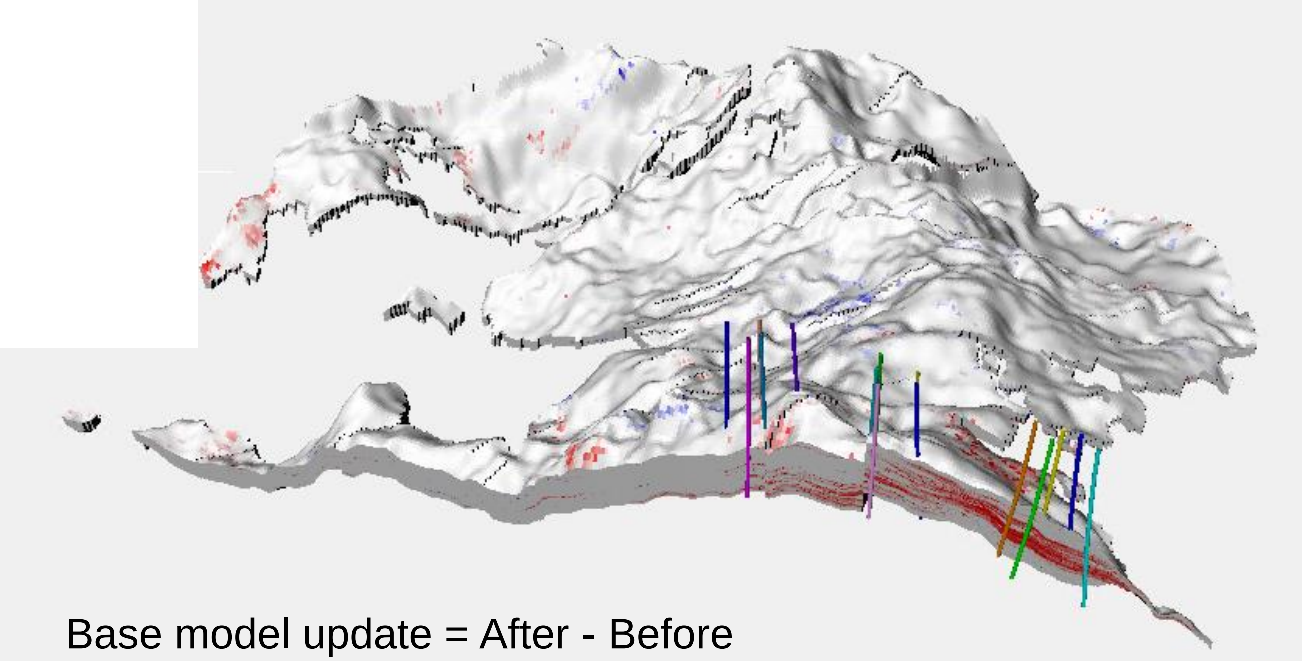
- Due to the embedment of the base model in the ensemble, the update from the base model is minimized and locally contained.



Base model (Real#1) before ESMDA



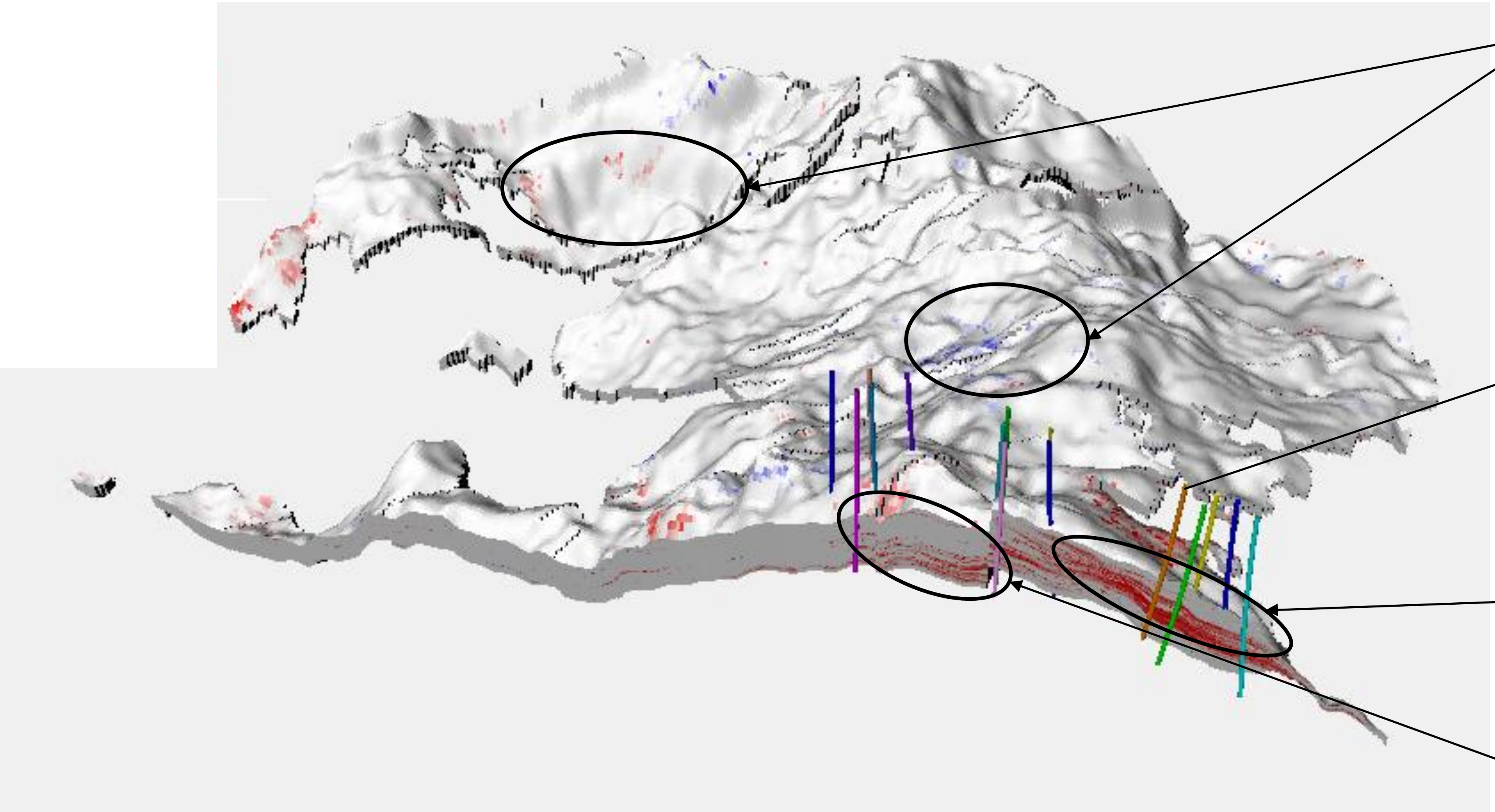
Base model (Real#1) after ESMDA update



Base model update = After - Before

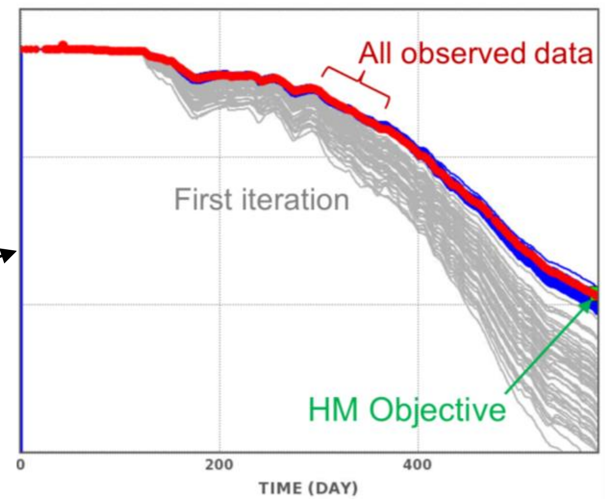
ESMDA for Deterministic History Matching

- Update is mostly confined to relevant area identified by ESMDA, localization working
- Possible spurious update could be alleviated by (a) Manually limit update to area of interest, (b) increase ensemble size



Base model update = After - Before

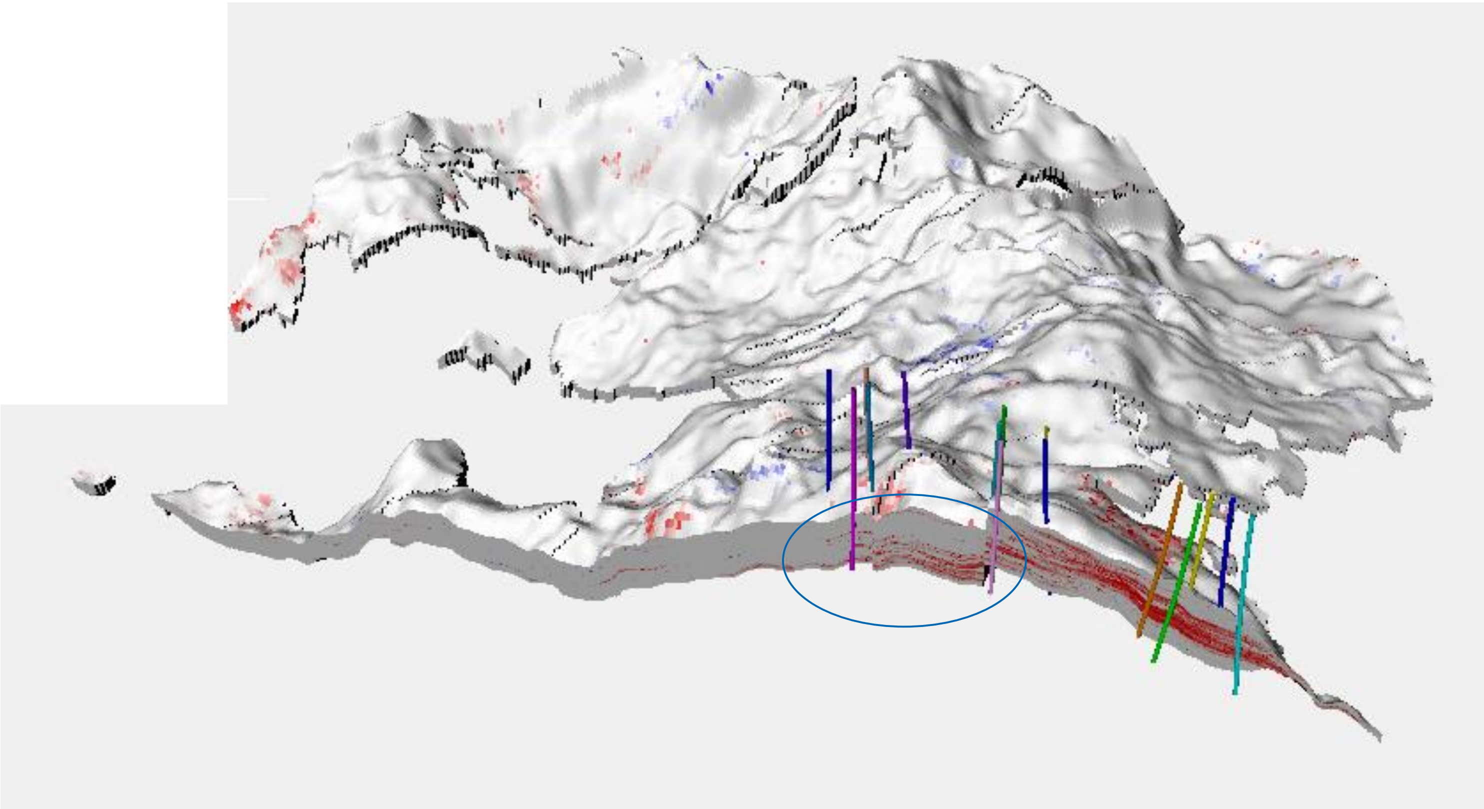
- Spurious update: possibly due to the generation of prior ensemble



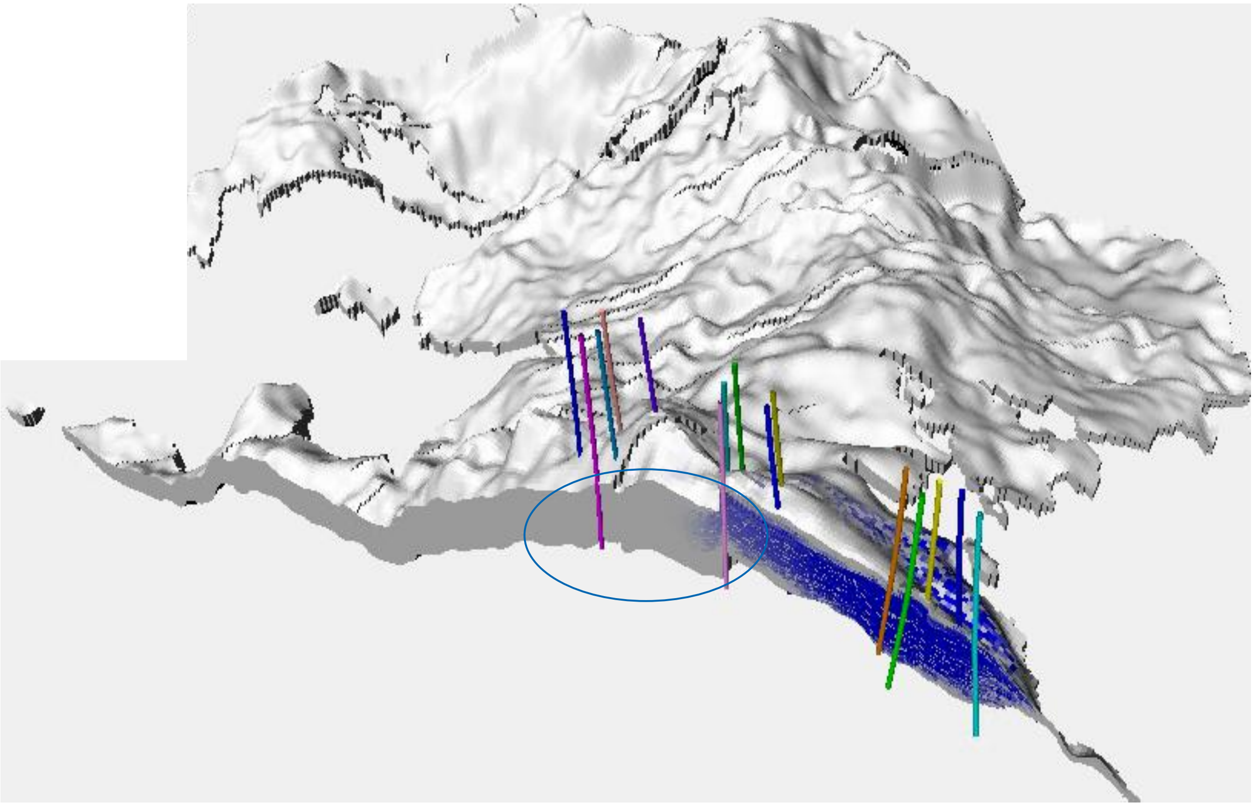
- Local update suggested by ESMDA to match target well BHP data
- Local update suggested by ESMDA based on correlation/continuity

Grid-Based Model Update from ESMDA vs Adjoint

- Both ESMDA and Adjoint identify similar areas to update
- ESMDA also identifies areas based on prior correlation/continuity, while Adjoint does not
- Adjoint is free of spurious update



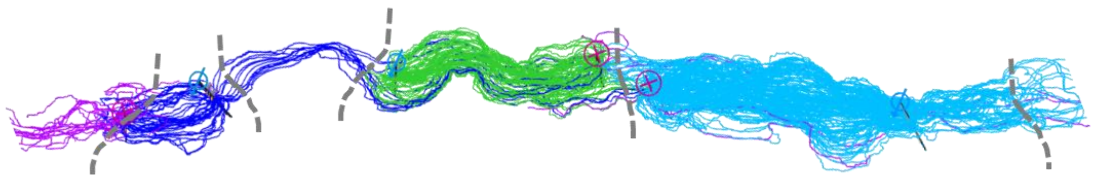
Porosity Update from ESMDA



Porosity update from Adjoint Gradient

Example 3: History Matching with Parameter-Based Uncertainty

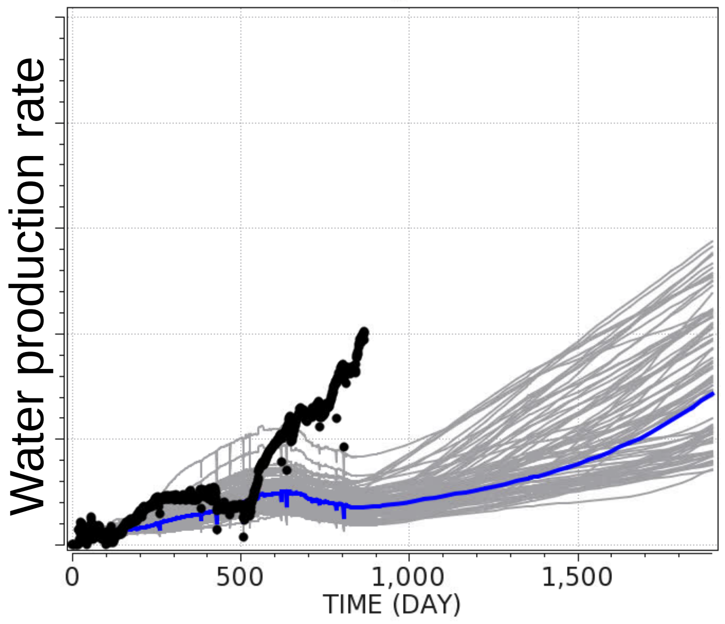
- Fluvial reservoir with parameter-based uncertainty characterization
- 86 regions with 546 uncertainties to match 3 years of data from 6 wells
  - Regional: k, kvkh, pv, cut-off; Global: PVT, initial GOR, fluid contacts
  - Facies: Rel. Perm.; Well/Completions: PI, PI degradation
  - Regions defined based on drainage area and seismic
- ESMDA was able to match data not covered by prior ensemble
- Uncertainty reduction makes sense qualitatively but not reliable quantitatively



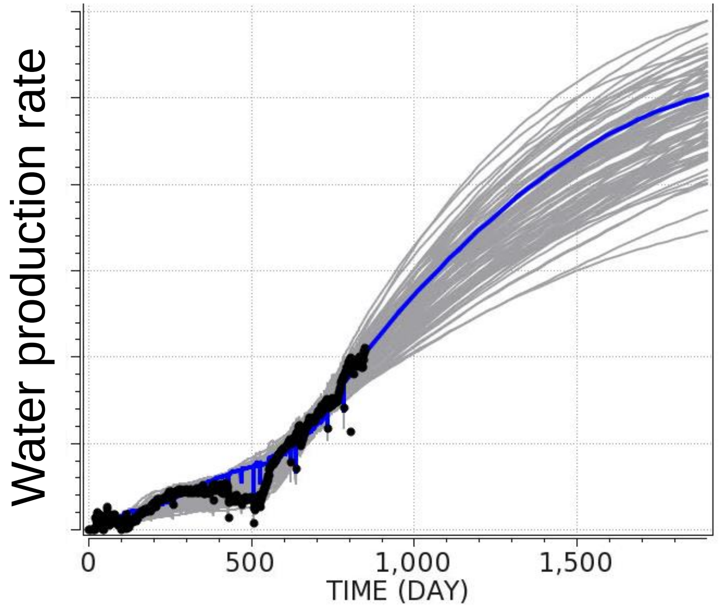
Drainage Region

Seismic inversion volume

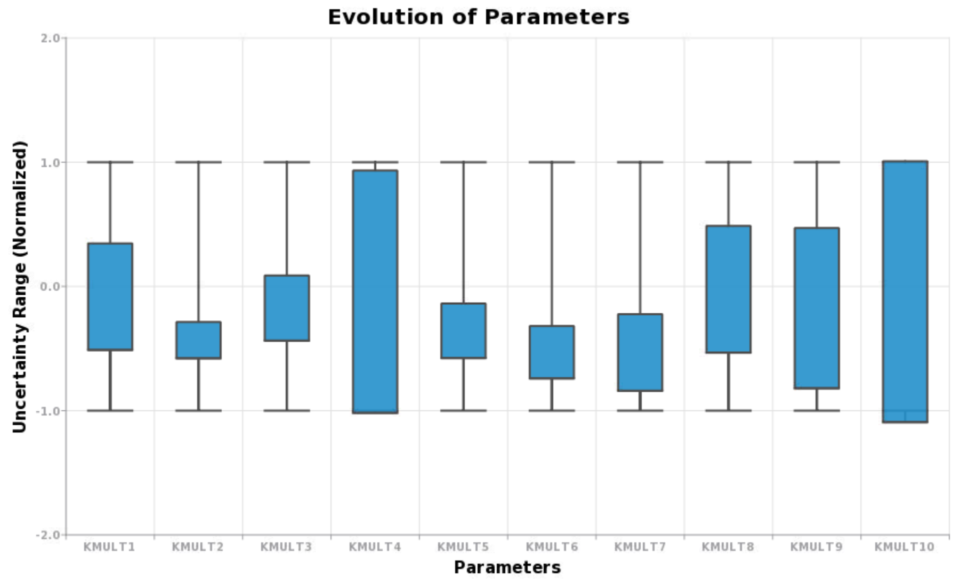
History matching regions



Before history matching



After history matching



Prior and posterior parameter distributions

Summary

- Lesson Learned:
  - ESMDA does not work well with categorical uncertainties
  - ESMDA can be applied for both parameter-based and realization-based uncertainty characterization
  - Dummy variable can be used to validate/invalidate the uncertainty quantification

- Challenge faced
  - When used for probabilistic history matching, ESMDA still suffer ensemble collapse even with localization in addition to nonlinearity of the problem, making it unreliable for uncertainty quantification
  - When used for deterministic history matching, it is an open question as to how to select one model

Outline

- History matching in oil and gas
- Implementation of ESMDA
- Probabilistic history matching example
- Deterministic history matching example
- Quantifying modeling error

Formulating the Error

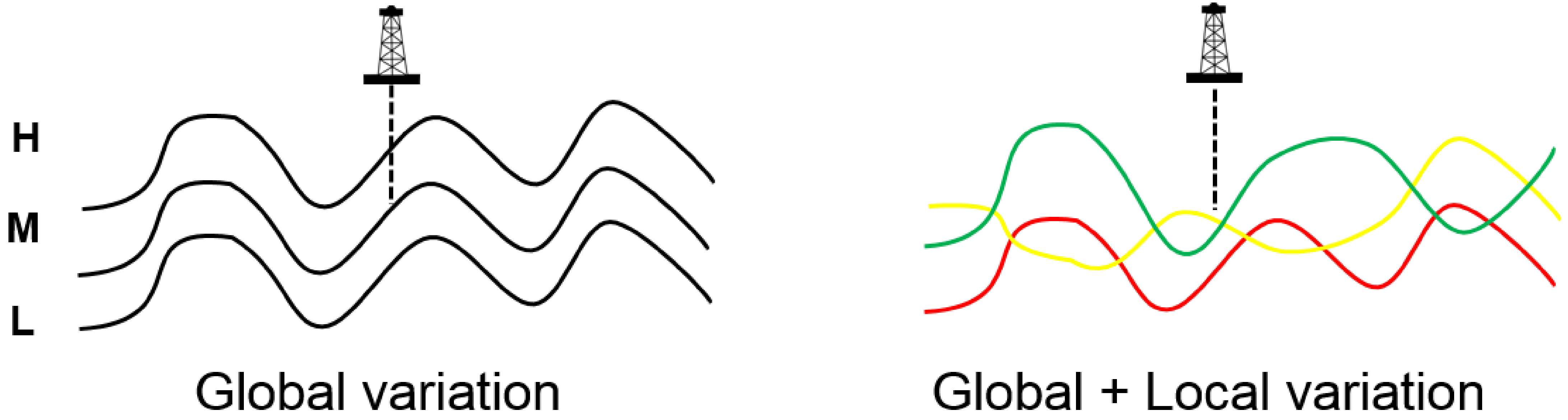
- Result of any probabilistic history matching method heavily depend on the error

$$\boldsymbol{m}_i^{n+1} = \boldsymbol{m}_i^n + \boldsymbol{C}_{md}^n(\boldsymbol{C}_{dd}^n + \alpha_n \boldsymbol{C}_e^n)^{-1}[\boldsymbol{d}_i^n - (\boldsymbol{d}_{obs}^n + \boldsymbol{e}_i^n)]$$

- Four concepts of data
  - Simulated data ($\tilde{\boldsymbol{d}}$), observation data ($\boldsymbol{d}$), true response ($\hat{\boldsymbol{d}}$) and observed data ($\boldsymbol{d}_{\boldsymbol{obs}}$)
  - Measurement error: $\mathbf{e}_{\boldsymbol{me}} = \boldsymbol{d} - \hat{\boldsymbol{d}}$
    - For example, gauge accuracy, indirect measurement
  - Modeling error: $\mathbf{e}_{\mathbf{mo}} = \hat{\boldsymbol{d}} - \tilde{\boldsymbol{d}}$
    - For example, uncaptured physics, missing small scale uncertainty
- Quantifying error
  - Physics-based: Identify the major sources of error and analyze them one-by-one
  - Data-driven: Quantify error based on inconsistency between observed data and simulated data
    - Cons: No inconsistency does not mean no error

Modeling Error due to Omission of Local Variation

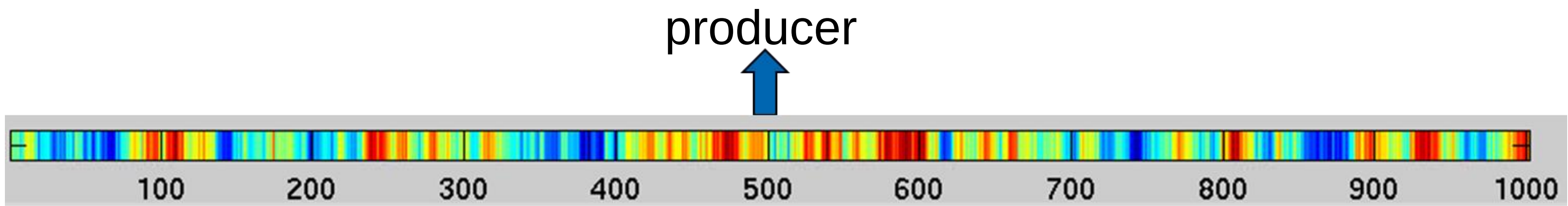
- Local and global variation:
  - Static uncertainty routinely characterized by long range uncertainty (multipliers)
  - Learning from local data may be falsely generalized to the full-field
  - This leads to overestimated S-curve update during history matching



He et al., Calibrating Global Uncertainties to Local Data: Is the Learning Being Over-Generalized? SPE ATCE 2018

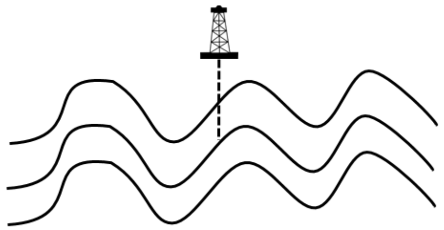
Example Problem

- 1D primary production problem (1001 grids)
  - Data: BHP at the producer at 100 days
  - Porosity field: $\boldsymbol{\phi} = \boldsymbol{\phi}_g + \boldsymbol{\phi}_l$
  - Objective function: Original oil in place ($OOIP = \sum_i \phi_i S_{oi} V_i$)

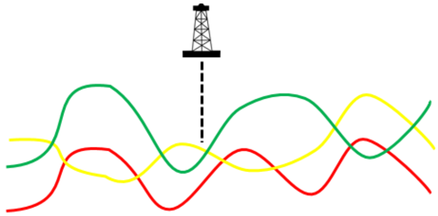


1D toy reservoir, porosity is shown

- Coarse uncertainty characterization (global only)
  - Global porosity multiplier ~N(0.25,0.03)



- Fine uncertainty characterization (global + local)
  - Global porosity multiplier ~N(0.25,0.03)
  - Local porosity variation ~N(0,0.0.5) & variogram=1500 ft



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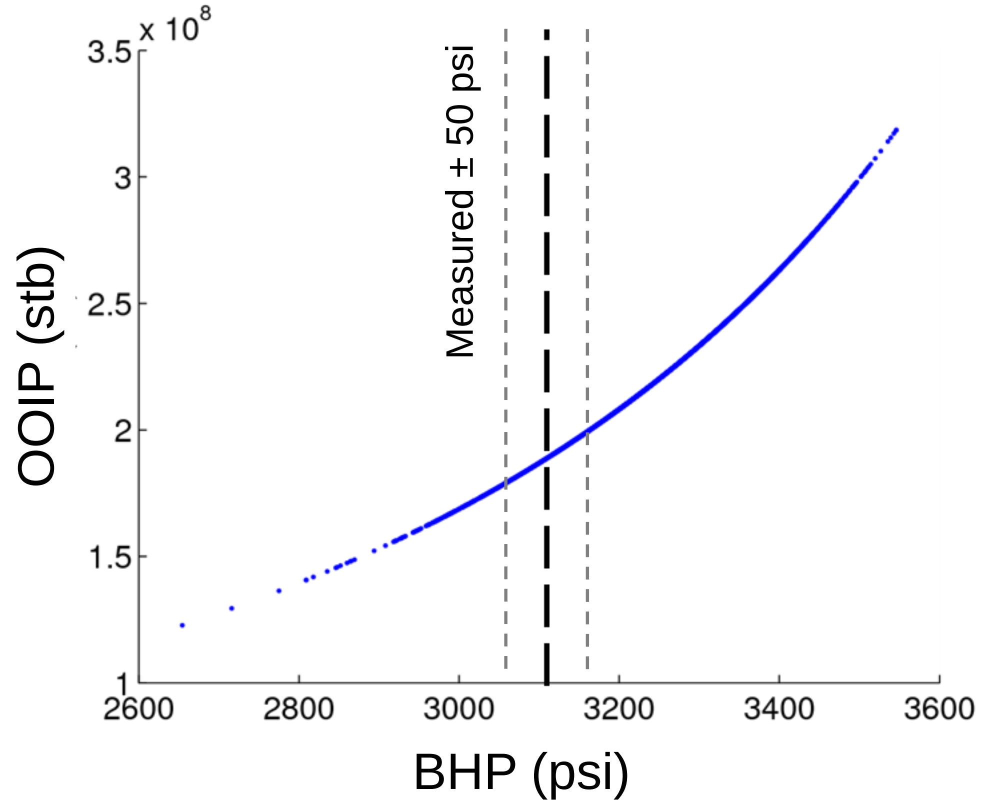
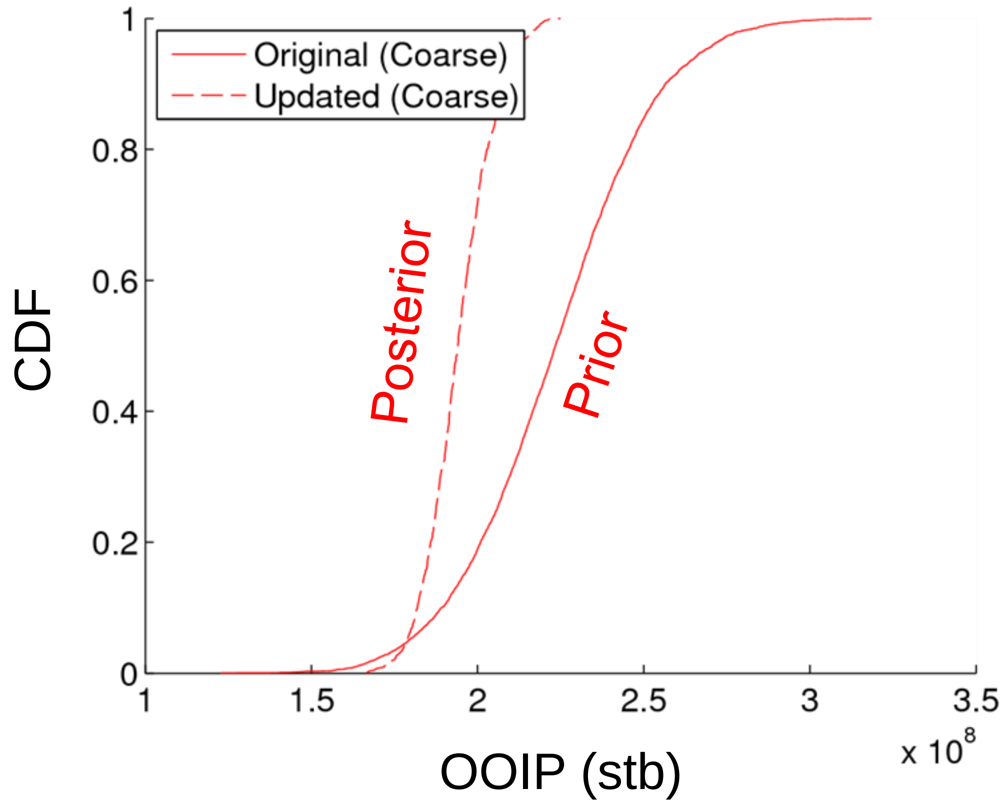
S-Curve Update with Coarse Model

- Rejection sampling is used with measured BHP (3100 psi)
- Little OOIP uncertainty left with 50 psi of measurement error





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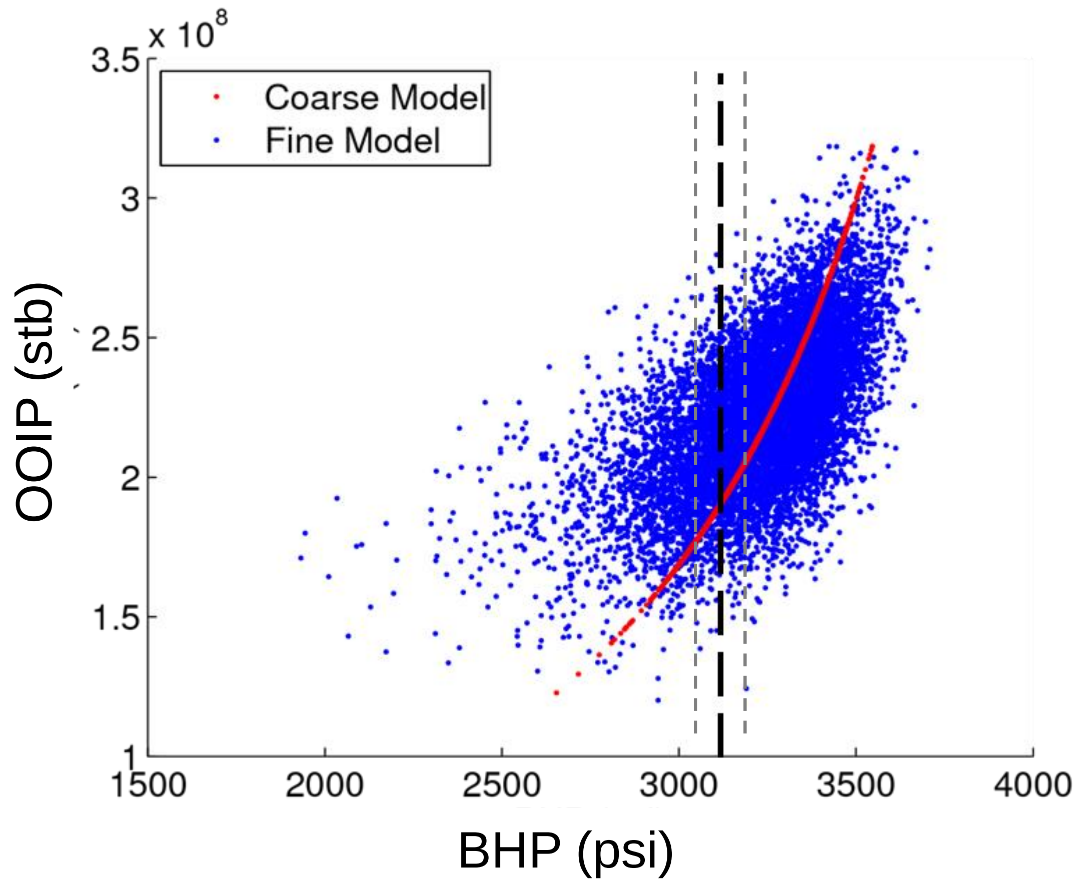
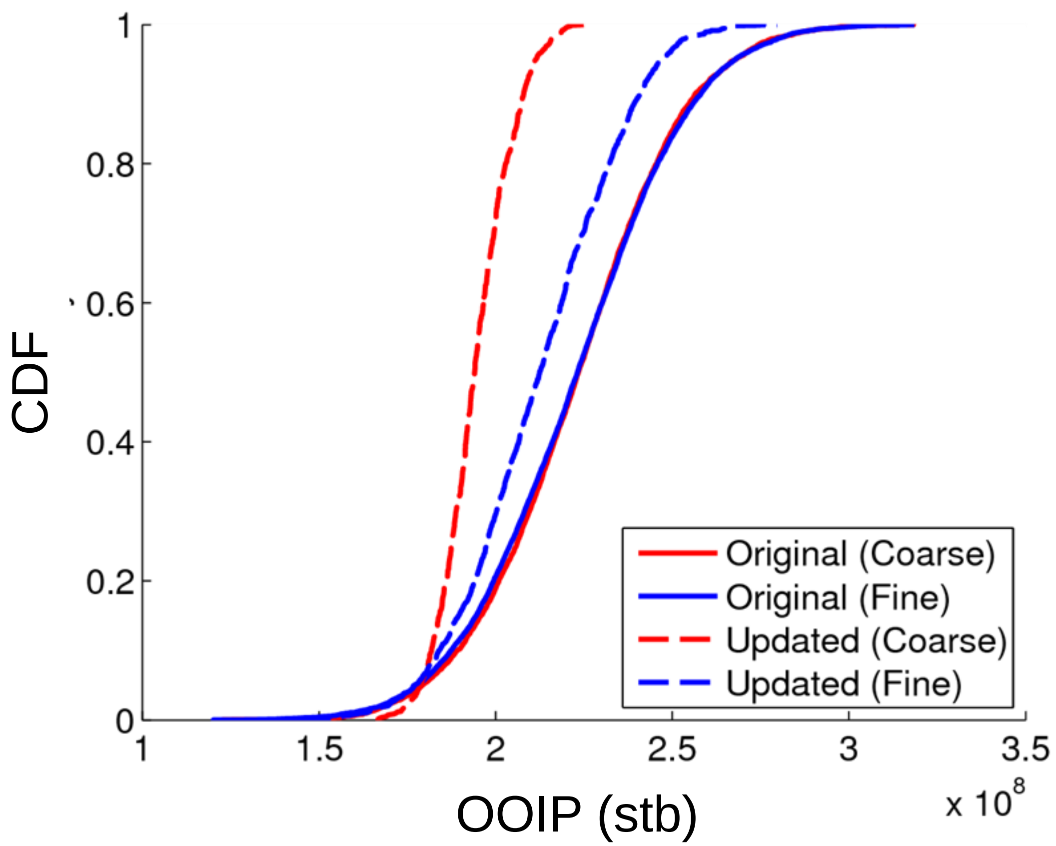
S-Curve Update with Fine Model

- Spread of data to OOIP substantially increased
- Local ϕ variation has no impact for prior OOIP CDF, while large impact for posterior distribution.





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Problem Formulation

- Assumption: Objective function J is global, not affected by local param. variation

$$J_f = J(\boldsymbol{m}_g + \boldsymbol{m}_l) = J(\boldsymbol{m}_g) = J_c = J$$

- Distribution of samples from fine characterization: $P(J, d_f)$
- Distribution of samples from coarse characterization: $P(J, d_c)$
- Bayes rule for posterior distribution $P((J, d)|d_{obs})$

$$P((J,d)|d_{obs}) = P(J,d)\frac{P(d_{obs}|(J,d))}{P(d_{obs})}$$

- Rejection sampling using coarse model

$$P_{acc} = \exp\left(-\frac{1}{2}(d - d_{obs})^T \Sigma_e^{-1}(d - d_{obs})\right)$$

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Modeling Error and Correction Factor

- Fine characterization data

$$d_{obs} = d_f + e_{me}$$

- Coarse characterization data

$$d_{obs} = d_c + e_{mo} + e_{me}$$

- Rejection sampling using fine model

$$P_{acc} = \exp\left(-\frac{1}{2}(d_f - d_{obs})^T \Sigma_{me}^{-1}(d_f - d_{obs})\right)$$

- Rejection sampling using coarse model

$$P_{acc} = \exp\left(-\frac{1}{2}(d_c - d_{obs})^T (\Sigma_{mo} + \Sigma_{me})^{-1}(d_c - d_{obs})\right)$$

- Modeling error and inflation factor τ

$$\Sigma_{mo} = (\Sigma_{dd})_f - (\Sigma_{dd})_c = \tau(\Sigma_{dd})_c$$

- Modeling error is proportional to variance in the simulated data

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Derivation for Correction Factor

- Analytical formula for correction factor τ

$$\tau_{d_1 d_2} = \frac{\int_\Omega \int_\Omega f_1(\mathbf{x}_i) C_l(\mathbf{x}_i, \mathbf{x}_j) f_2(\mathbf{x}_j) d\mathbf{x}_i d\mathbf{x}_j}{\int_\Omega \int_\Omega f_1(\mathbf{x}_i) C_g(\mathbf{x}_i, \mathbf{x}_j) f_2(\mathbf{x}_j) d\mathbf{x}_i d\mathbf{x}_j}$$

$f_i(x)$: How sensitive is data d_i to uncertain properties at location x

$C(x_i, x_j)$: How correlated are uncertain properties at location x_i and x_j

- Empirical formula from regression of numerical solution

$$\tau_{dd} = \frac{\sigma^2_{m,l}}{\sigma^2_{m,g}} \left(\frac{1}{2(\gamma+1)^2} + \frac{1}{2(\gamma+1)} \right)$$

Formula for 1D problem

$$\tau_{dd} = \frac{\sigma^2_{m,l}}{\sigma^2_{m,g}} \exp\left(-0.8 \left(\frac{\lambda}{\rho} \right)^{0.9} \right)$$

Formula for 2D isotropic problem

$$\tau_a = \frac{\sigma^2_{m,l}}{\sigma^2_{m,g}} \left(\frac{1}{0.4\gamma_x + 1} \right)^{1.5} \left(\frac{1}{0.4\gamma_y + 1} \right)^{1.5}$$

Formula for 2D anisotropic problem

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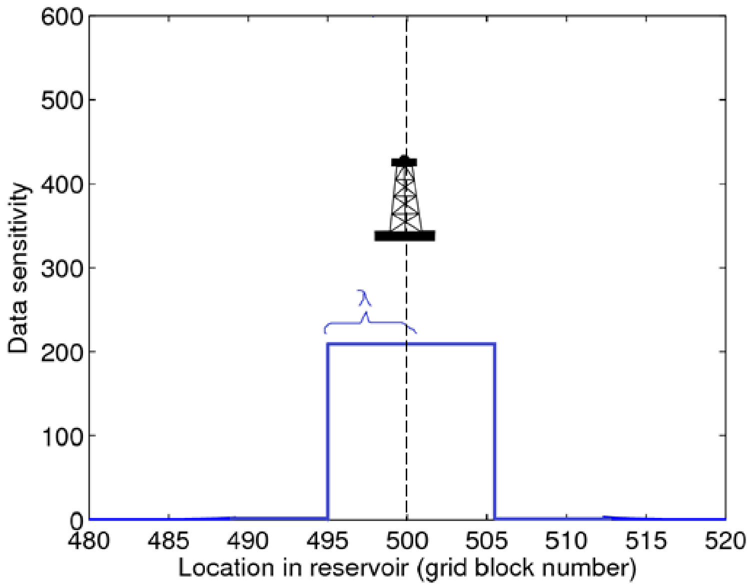
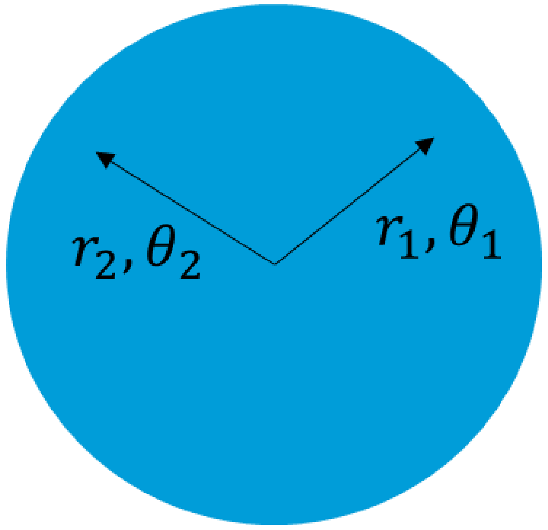
What Controls Correction Factor

- Formula for 2D isotropic problem from regression of numerical solutions

$$\tau_{dd} = \frac{\sigma^2_{m,l}}{\sigma^2_{m,g}} \exp\left(-0.8\left(\frac{\lambda}{\rho}\right)^{0.9}\right)$$

- Strength of local variation compared with the global variation $\frac{\sigma^2_{m,l}}{\sigma^2_{m,g}}$
  - Strong local variation -> large modeling error
- Data detection range λ:
  - Small λ -> large modeling error
- Variogram range ρ:
  - Small ρ -> small modeling error
  - <u>Local assumption: ρ << reservoir size</u>





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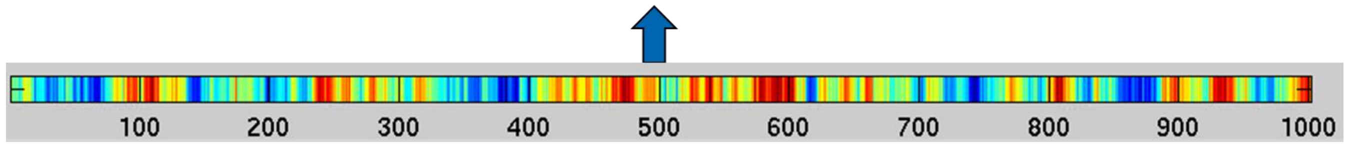
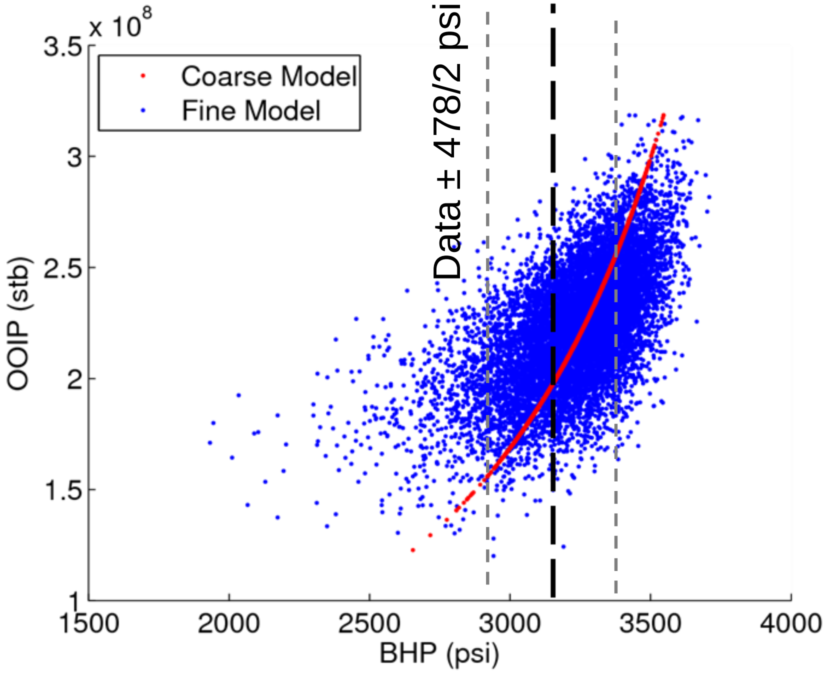
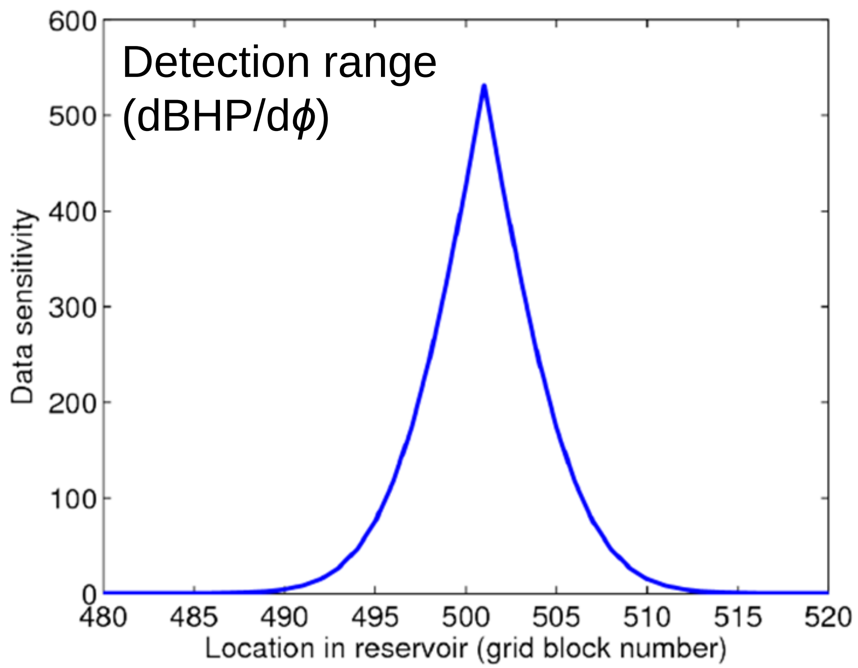
Case 1. Single Well Measurement with 1D Problem

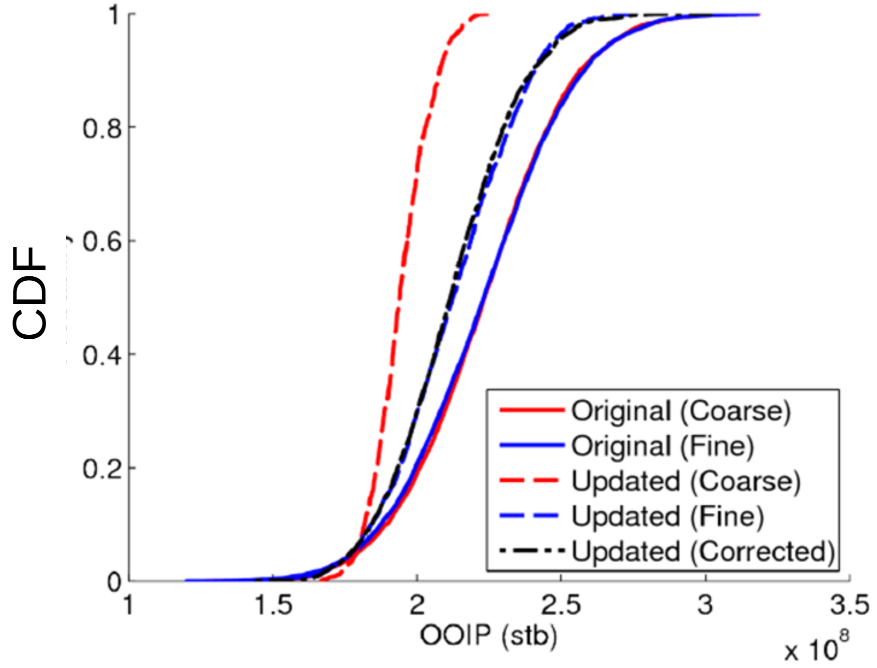
- Measuring well BHP to calibrate OOIP
  - Global porosity component: $\sigma_{\phi,c} = 0.03$
  - Local porosity component: $\sigma_{\phi,f} = 0.05;\ \rho = 4000ft$
  - Detection range: $\lambda = 1500ft$, simulation BHP in std: $\Sigma_{dd,c}$= 280 psi



- Modeling error $\frac{\sigma^2_{m,l}}{\sigma^2_{m,g}}\left(\frac{1}{2(\gamma+1)^2}+\frac{1}{2(\gamma+1)}\right)\Sigma_{dd,c}$ is 478 psi in std.







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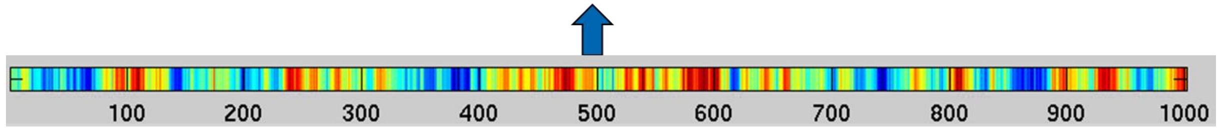
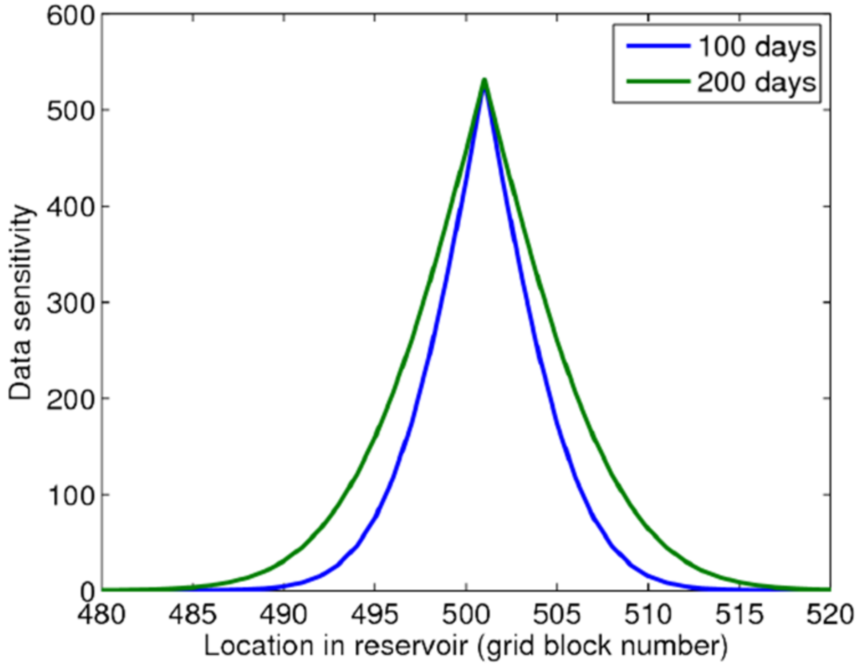
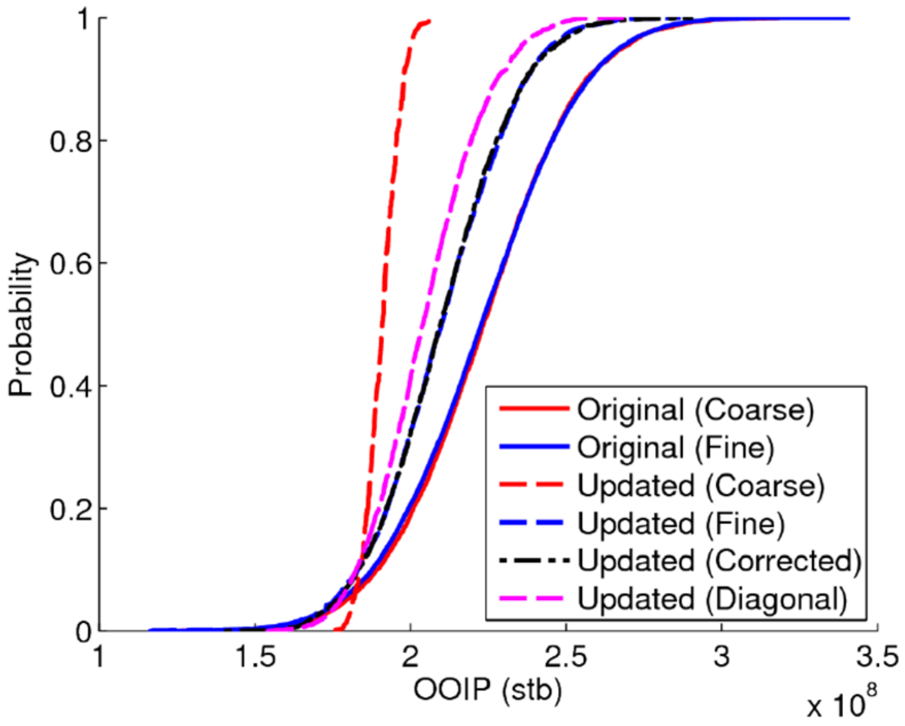
Case 2. Single Well Consecutive Measurement

- Measuring well BHP at 100 day and 200 day
  - Global porosity component: $\sigma_{\phi,c} = 0.03$
  - Local porosity component: $\sigma_{\phi,f} = 0.05;\ \rho = 4000ft$
- Modeling error for the two data points are <u>highly correlated</u>





$$\Sigma_{mo} = \begin{bmatrix} 19388 & 25180 \\ 25180 & 33049 \end{bmatrix}$$



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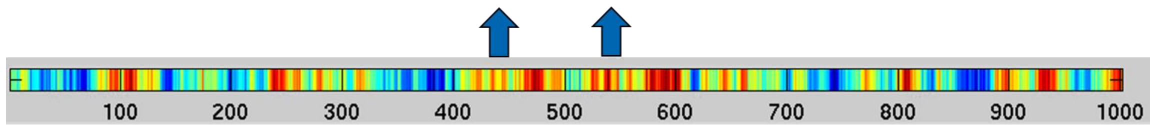
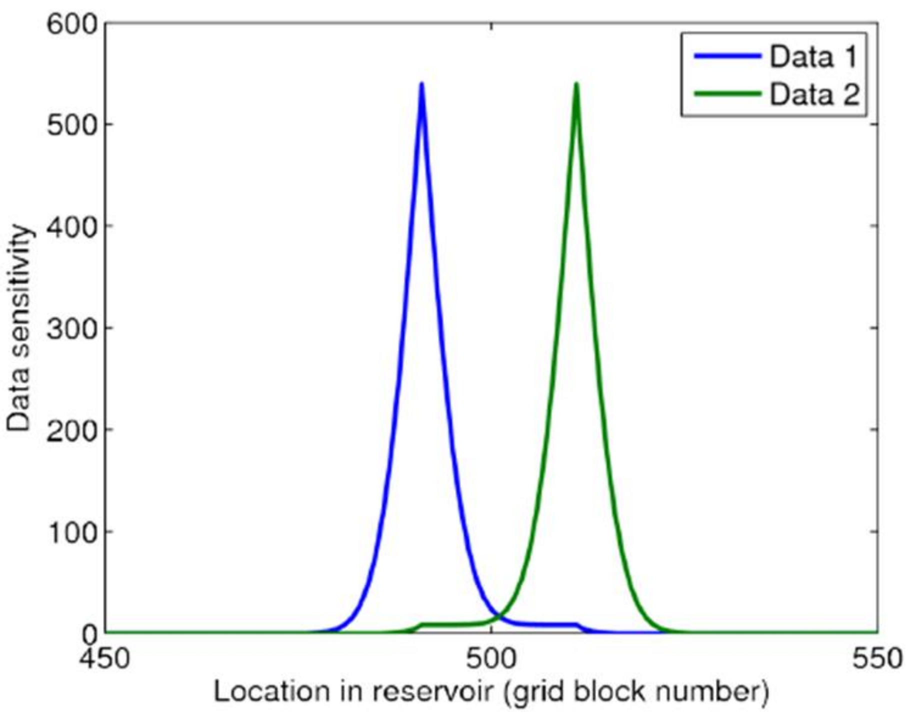
Chevron

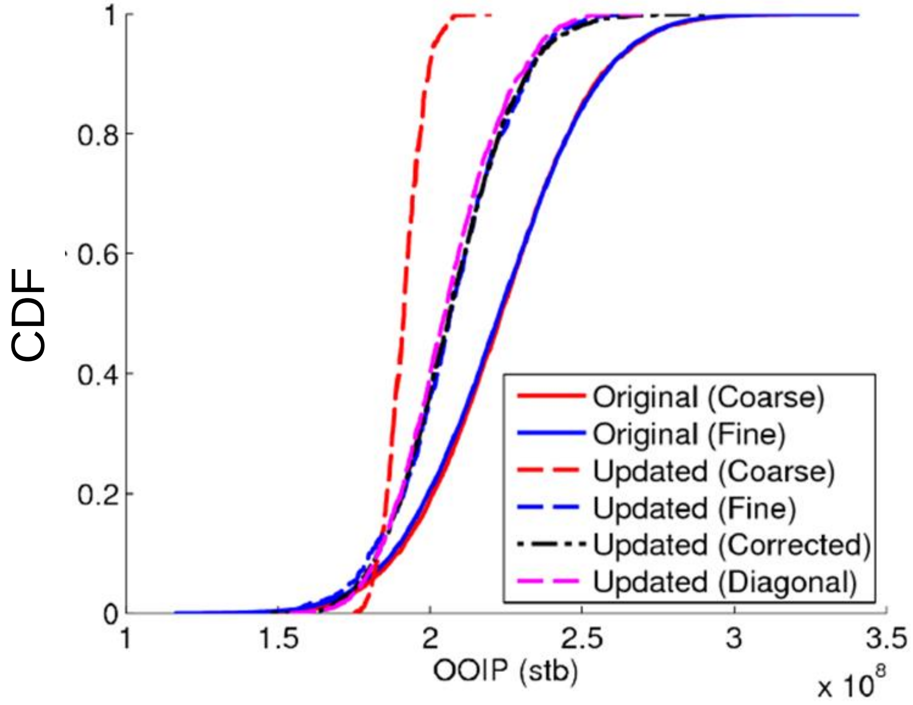
Case 3. Measurements from Two Different Wells

- Measuring well BHP at 100 day and 200 day
  - Global porosity component: $\sigma_{\phi,c} = 0.03$
  - Local porosity component: $\sigma_{\phi,f} = 0.05$; $\rho = 4000ft$
- Modeling error for the two data points are weakly correlated





$$\Sigma_{mo} = \begin{bmatrix} 21518 & 4161 \\ 4161 & 21518 \end{bmatrix}$$



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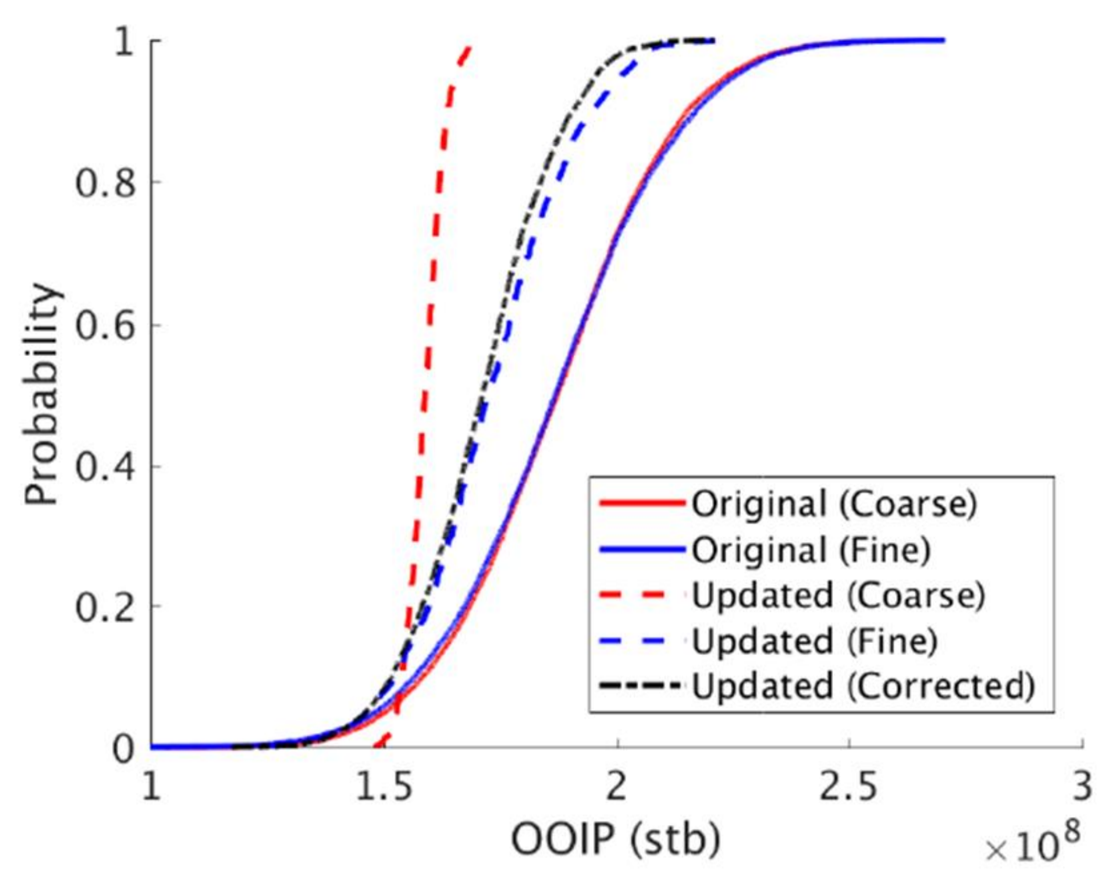
Case 4. 2D Isotropic Variogram Example

- Global porosity component: $\sigma_{\phi,c} = 0.03$
- Local porosity component: $\sigma_{\phi,f} = 0.05$; $\rho_x = \rho_y = 1500ft$
- Data detection range: $\lambda = 2500ft$

$$\tau_{dd} = \frac{\sigma^2_{m,l}}{\sigma^2_{m,g}} \exp\left(-0.8\left(\frac{\lambda}{\rho}\right)^{0.9}\right)$$

Formula for 2D isotropic problem



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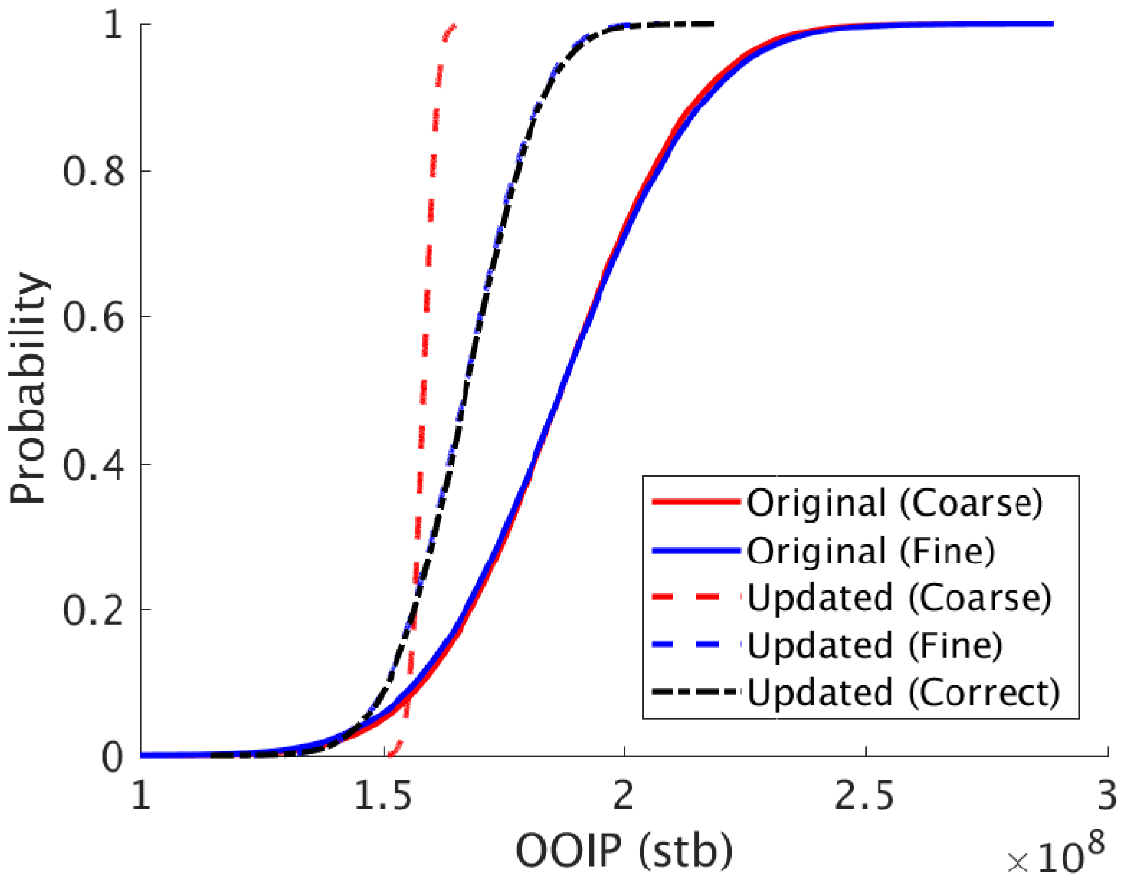
Case 5. 2D Anisotropic Variogram Example

- Global porosity component: $\sigma_{\phi,c} = 0.03$
- Local porosity component: $\sigma_{\phi,f} = 0.05$; $\rho_x = 2000ft$; $\rho_y = 1500ft$
- Data detection range: $\lambda_x = 10000ft; \lambda_y = 5000ft$

$$\tau_a = \frac{\sigma^2_{m,l}}{\sigma^2_{m,g}} \left(\frac{1}{0.4\gamma_x + 1}\right)^{1.5} \left(\frac{1}{0.4\gamma_y + 1}\right)^{1.5}$$

Formula for 2D anisotropic problem



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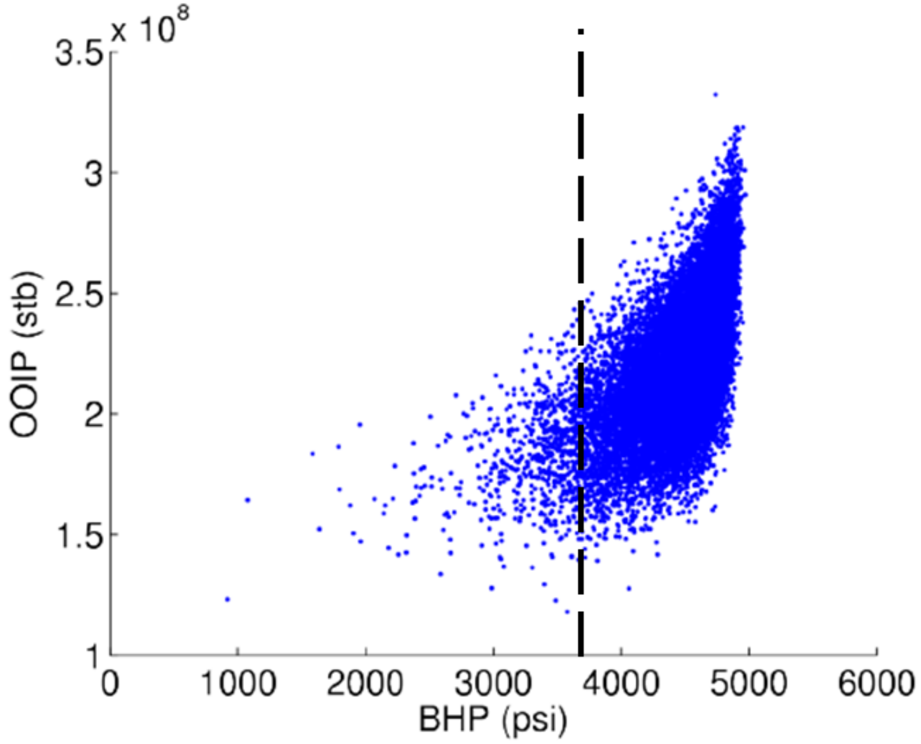
Case 6. Extension to Multiple Random Fields

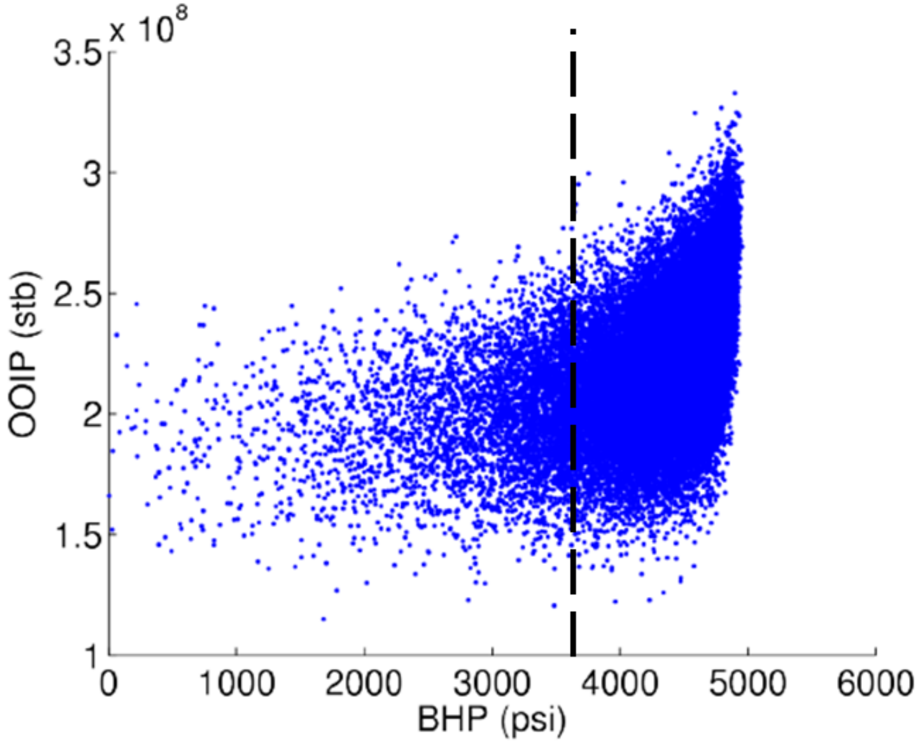
- Correction factor approximated as convex combination of those for individual fields

$$\hat{\tau}_{total} = w_\phi \tau_\phi + w_\zeta \tau_\zeta$$

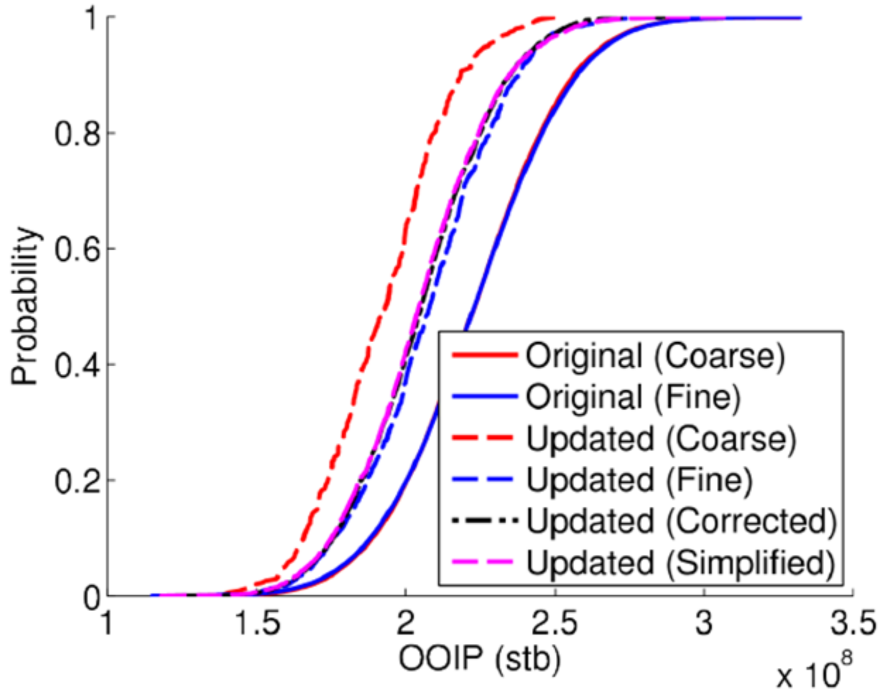
- Random field for porosity: Global ~ $N(0.25, 0.03)$; Local ~ $N(0, 0.04)$, $\rho = 4000ft$
- Random field for permeability: Cloud transform from porosity with 0.71 correlation



(a) Coarse uncertainty characterization



(b) Fine uncertainty characterization



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Summary

- Calibrate of global objective function with local data when omitting of local variation leads to over-estimated S-curve update
- The error incurred by this local-global effect depends on
  - Variance in the simulated data
  - Data detection range
  - Ratio between variances of local and global variation
  - Variogram range of the local variation
- Local-global modeling error can be highly correlated for different data points
- Formula for correction factor is proposed and validated

$$\tau_{dd} = \frac{\sigma^2_{m,l}}{\sigma^2_{m,g}}\left(\frac{1}{2(\gamma+1)^2} + \frac{1}{2(\gamma+1)}\right)$$

Formula for 1D problem

$$\tau_{dd} = \frac{\sigma^2_{m,l}}{\sigma^2_{m,g}} \exp\left(-0.8\left(\frac{\lambda}{\rho}\right)^{0.9}\right)$$

Formula for 2D isotropic problem

$$\tau_{a} = \frac{\sigma^2_{m,l}}{\sigma^2_{m,g}}\left(\frac{1}{0.4\gamma_x+1}\right)^{1.5}\left(\frac{1}{0.4\gamma_y+1}\right)^{1.5}$$

Formula for 2D anisotropic problem

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Summary

- Strong interest from field engineers to use ensemble method for data assimilation
- Various types of problems
  - Parameter-based vs realization-based uncertainty characterization
  - Probabilistic vs deterministic history matching
- Final ensemble from ESMDA not always reliable for uncertainty quantification, even with localization
- Still an open question how to select the best posterior model in the deterministic history matching setting
- Parameter-based uncertainty characterization that omits local variation could lead to model error
  - Such error is proportional to the data variance in the coarse characterization
  - Such error could be highly correlated for different data points
  - Such error could be estimated and corrected through numerical/empirical formula