Using Bayesian Model Probability with ensemble methods to quantify uncertainty in reservoir modelling with multiple prior scenarios

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Outline

- Motivation
- Introduction to Bayesian model averaging and selection
  - Bayesian Model Average (BMA)
  - Bayesian Model Probability (BMP)
  - Model Likelihood/Model Evidence (BME)
  - Bayes Factor (BF)
- Calculating BME/BMP
  - Challenges
- Examples
- Summary and conclusions

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  - Geological scenarios, flow scenarios, alternative seismic interpretations, etc.
- Ensemble-based data assimilation methods, like EnKF or ES, does not handle alternative prior models or scenarios. Handling complex model uncertanty is challenging.
- Bayesian theory for models provides a framework for this:
  - "Total" uncertainty through Bayesian Model Averaging (BMA).
  - Selecting models or scenarios based on comparing Bayesian Model Probabilities (BMP's), i.e., probability for a given model or scenario to be correct given the data.

Motivation, cont'd

- Bayesian model averaging and model probability rely on the calculation of Bayesian Model Evidence (BME) or Bayes Factors (BF), which have a long history within a number of fields for model comparison and model selection.

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- Recently, it has been shown that, for weakly nonlinear models, Bayesian model selection can be efficiently coupled with ensemble-based data assimilation methods (Carrassi et al., 2017).
- The methodology can be applied as a "simple" post-processing after having applied standard ensemble-based data assimilation methods to the various scenarios.

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- Recently, it has been shown that, for weakly nonlinear models, Bayesian model selection can be efficiently coupled with ensemble-based data assimilation methods (Carrassi et al., 2017).
- The methodology can be applied as a "simple" post-processing after having applied standard ensemble-based data assimilation methods to the various scenarios.
- However, the use of these methods are disputed. The calculations may be very challenging with respect to e.g. stability, and further investigations of the applicability to reservoir modeling and updating are necessary.

BMA/BMP/BME/BF

- Bayesian Model Average (BMA):

$$P\left(\Delta|D\right) = \sum_k P(\Delta|D, M_k)P(M_k|D)$$

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- Bayes factor (BF):

$$BF_{j\text{-}k} = \frac{P(D|M_j)}{P(D|M_k)}$$

Some alternatives for calculating BME

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\begin{aligned}
P(D|M_k) &= \mathcal{N}(G_k\overline{\theta}_k, C_k) \\
&= \left((2\pi)^{n_d} \det C_k\right)^{-1/2} \exp\left\{-\frac{1}{2}(D - G_k\overline{\theta}_k)^T C_k^{-1}(D - G_k\overline{\theta}_k)\right\} \\
C_k &= C_D + G_k C_{\theta k} G_k^T
\end{aligned}
$$

$$
\frac{P(D|M_j)}{P(D|M_k)} = \left(\frac{\det C_k}{\det C_j}\right)^{1/2} \exp\left\{ -\frac{1}{2} \left[(D - G_j\overline{\theta}_j)^T C_j^{-1}(D - G_j\overline{\theta}_j) - (D - G_k\overline{\theta}_k)^T C_k^{-1}(D - G_k\overline{\theta}_k)\right]\right\}
$$

where $\overline{\theta}_k$ is prior mean and C_k is prior covariance matrix.

Utilizing the ensemble representation of the pdf's, the calculations may be performed in a space of dimension equal to the ensemble size.

Some alternatives for calculating BME

1. Gauss-linear approximation
2. "Inverted Bayes", i.e.,

$$P\left(D|M_k\right) = \frac{P(D|\theta, M_k)P(\theta|M_k)}{P(\theta|D, M_k)}$$

   using e.g. posterior mean or MAP estimate for θ.

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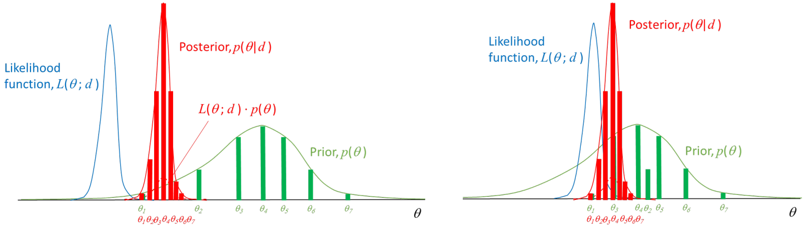

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4. Harmonic average of likelihoods over posterior ensemble (approximation to the importance sampling above):

$$P\left(D|M_k\right) \approx \left\{\frac{1}{N_e}\sum_{i=1}^{N_e}\frac{1}{P(D|\theta_i, M_k)}\right\}^{-1}, \quad (2)$$

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5. Eq. 2 with only one realization, i.e., simply the likelihood function (with posterior mean, e.g.).

Challenges

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- Likelihood function: Does not take into account prior uncertainty.

Some approaches to handle these challenges

- Multidimensional scaling and kernel density estimation of likelihood (Park et al., 2013).
- Localization (Metref et al., 2018).
- Multilevel methods (Hoel et al., 2016, Fossum et al., 2019)

Examples

From S.I. Aanonsen, S. Tveit and M. Alerini:

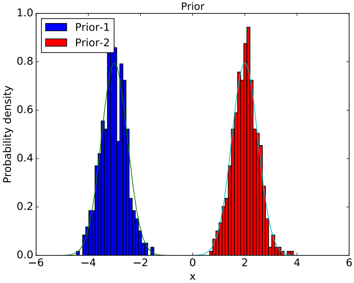
Using Bayesian Model Probability for Ranking Different Prior Scenarios in Reservoir History Matching

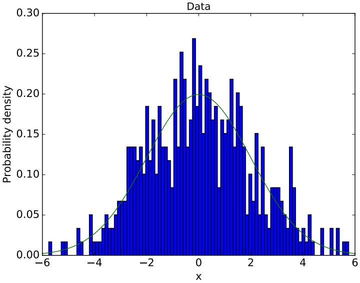
SPEJ, 2019

Example 1

Measurements of one, single parameter

Bimodal prior



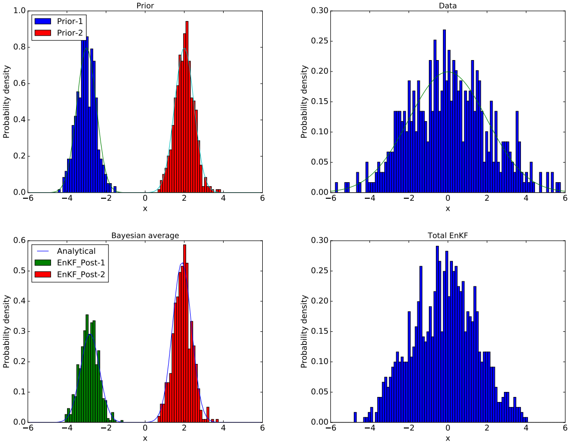


Prior: Mean = -3.0 / 2.0

Data: Mean = 0.0, Std = 2.0

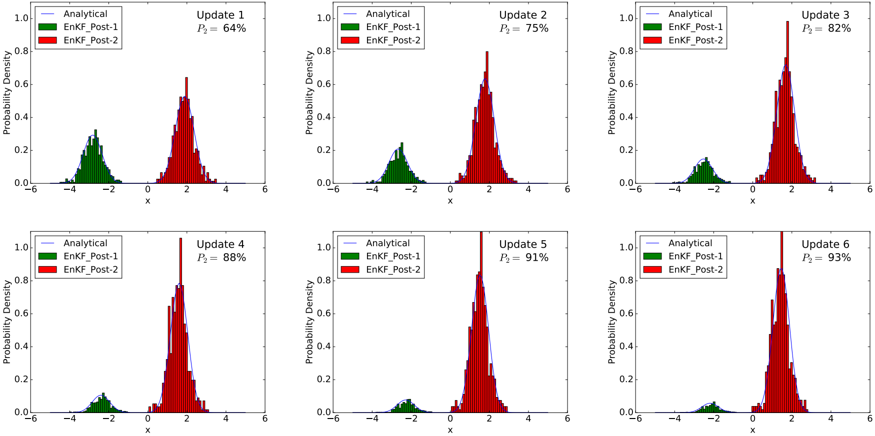
Example 1

Bayesian average vs "Total EnKF"



Example 1

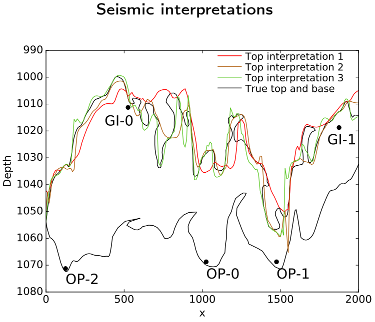
Bayesian average with repeated measurements

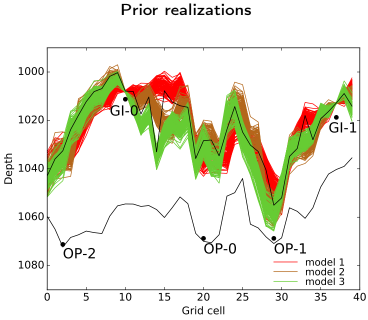


Example 2

Estimating top reservoir surface from 4D seismic data

Synthetic model inspired by a real North Sea oil field



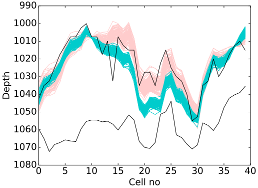


Gas injection into undersaturated oil reservoir. 2D cross section
Data: Gas-cap thickness
No of parameters: 40
No of ensemble members: 100

Example 2

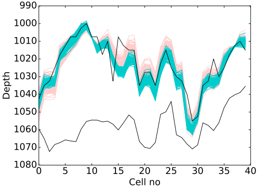
Results Ensemble Smoother. Data at 100 and 200 days

Model 1



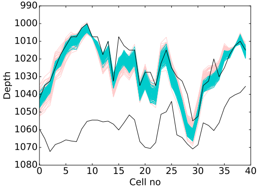
Data variance $9m^2$

Model 2



Data variance $9m^2$

Model 3



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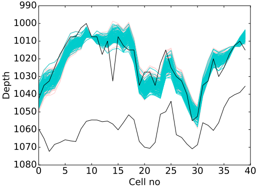
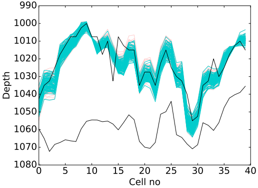


Data variance $360m^2$

Model 2



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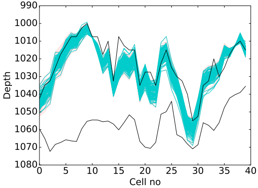

Data variance $360m^2$

Pink: Prior ensemble. Green: Posterior ensemble. Black: True model.

Example 2

Predicted GOC at 200 days. Data at 100 and 200 days

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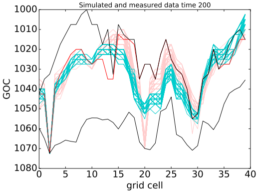

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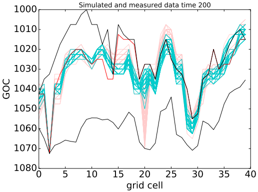

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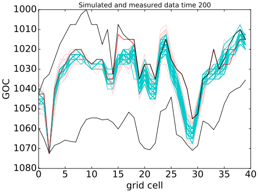

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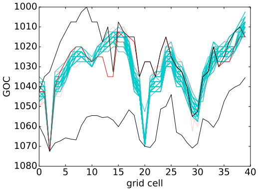

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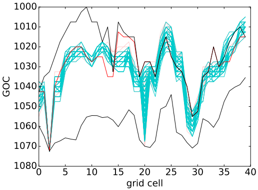

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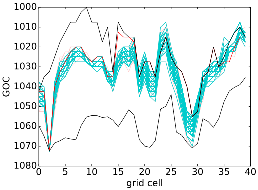

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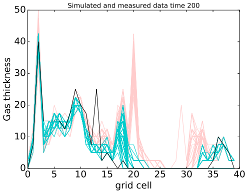
Pink: Predicted from prior. Green: Predicted from posterior. Red: True model.

Example 2

Predicted gas thickness at 200 days. Data at 100 and 200 days

Model 1



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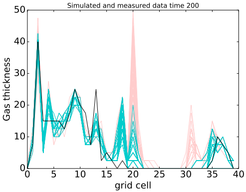
Model 2



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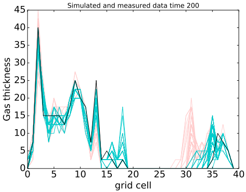

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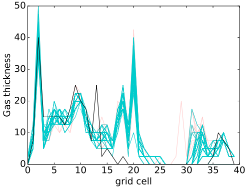

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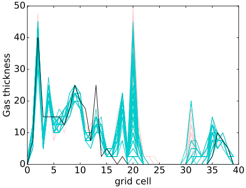

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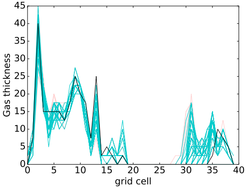

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Example 2

Model probability — effect of data variance

Data variance	Model	Gauss-Linear	Inverted Bayes*)	Importance sampling	Harmonic average
	1	1.4E-39	1.6E-62	1.4E-22	7.9E-24
$9m^2$	2	2.6E-15	1.4E-28	3.0E-12	6.3E-27
	3	1.0	1.0	1.0	1.0
	1	0.04	0.03	0.03	0.04
$360m^2$	2	0.15	0.11	0.15	0.11
	3	0.81	0.86	0.82	0.85

*) With $\theta =$ posterior mean

All pdf's are assumed Gaussian. Mean and covariances for prior and posterior from respective ensembles.

Example 2

Model probability — effect of the amount of data

Data	Model	Model probability
One seismic survey at 100 days	1	0.25
	2	0.33
	3	0.42
Two seismic surveys at 100 and 200 days	1	0.03
	2	0.15
	3	0.82
Ten seismic surveys at 100, 200,..., 1000 days	1	0.004
	2	0.01
	3	0.98

Data variance: $360m^2$
Method: Importance sampling

Example 2

Model probability (%) — 10 independent runs

	Mean	Std
Model 1	3.96	0.54
Model 2	18.3	2.28
Model 3	77.8	2.64

10 independent prior ensembles
Data: 2 surveys at 100 and 200 days
Data variance: $360m^2$
Method: Importance sampling

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- More research is needed to understand the methodology and its applicability to reservoir modelling.

Acknowledgments

Equinor ASA

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