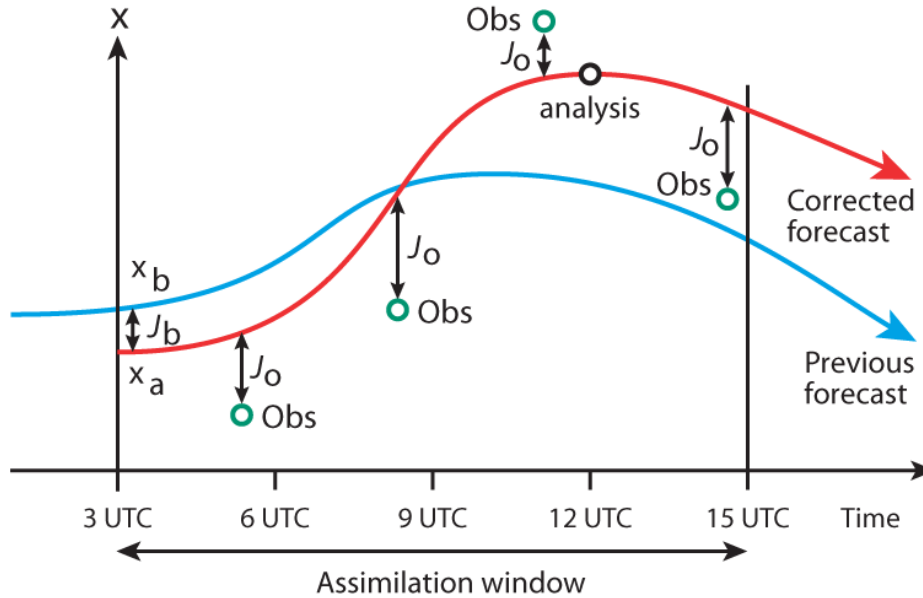


Bias-aware assimilation methods for numerical weather prediction

Patrick Laloyaux, Massimo Bonavita, Marcin Chrust, Mohamed Dahoui, Selime Gürol, Sean Healy, Elias Holm

4D-Var theory



4D-Var assumes **random zero-mean errors** for all sources of information used in the analysis

→ Is it correct to assume that NWP models are unbiased?

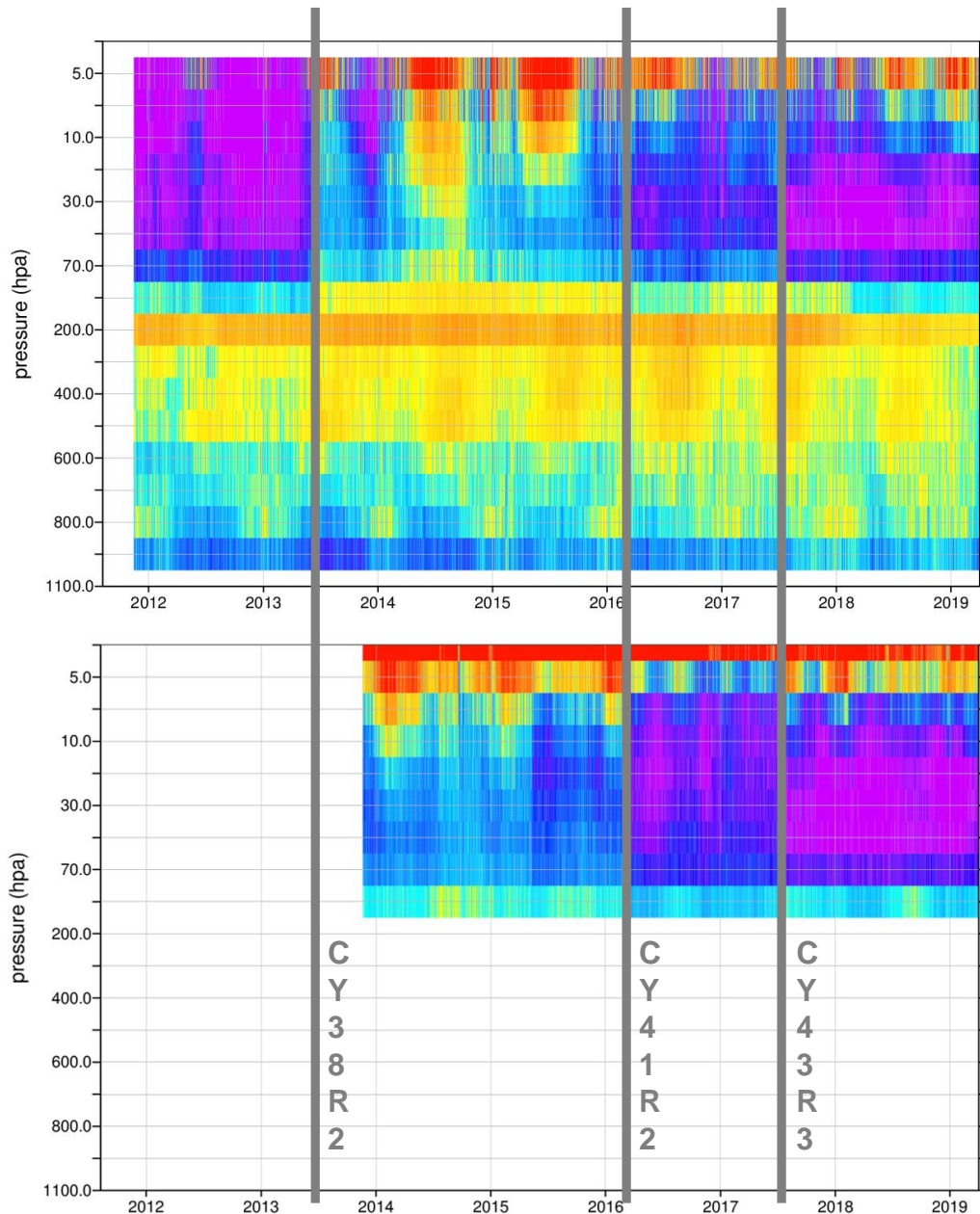
Cost function depends only on the state at the beginning of the assimilation window

$$J_{SC}(\mathbf{x}_0) = \frac{1}{2} (\mathbf{x}_0 - \mathbf{x}_0^b)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_0^b) + \frac{1}{2} \sum_{k=0}^N (\mathcal{H}_k \mathcal{M}_{k,0}(\mathbf{x}_0) - \mathbf{y}_k)^T \mathbf{R}_k^{-1} (\mathcal{H}_k \mathcal{M}_{k,0}(\mathbf{x}_0) - \mathbf{y}_k)$$

Equivalent to the standard Kalman Filter analysis update (linear operators)

$$\mathbf{x}_0 = \mathbf{x}_0^b + \left(\mathbf{B}^{-1} + \sum_{k=0}^N \mathbf{M}_{k,0}^T \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{M}_{k,0} \right)^{-1} \sum_{k=0}^N \mathbf{M}_{k,0}^T \mathbf{H}_k^T \mathbf{R}_k^{-1} (\mathbf{y}_k - \mathbf{H}_k \mathbf{M}_{k,0} \mathbf{x}_0^b)$$

Biases in the ECMWF atmospheric model (1/2)



The short-term model bias is estimated by comparing the 12-hour first-guess trajectory with radiosondes (181m of obs) and GPS-RO (123m of obs)

Similar signal with the two types of observations:

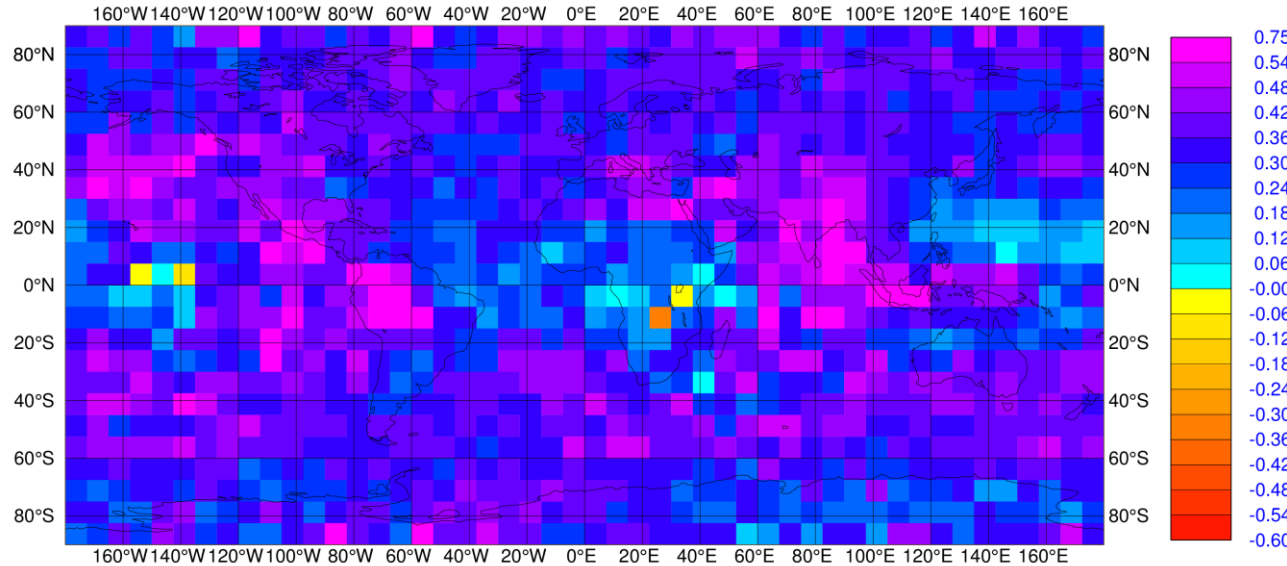
→ bias reduced with new vertical resolution (L137 in CY38R2)

→ bias increased with new horizontal resolution (Tco1279 in CY41R2)

→ bias increased with new radiative scheme (CY43R3)

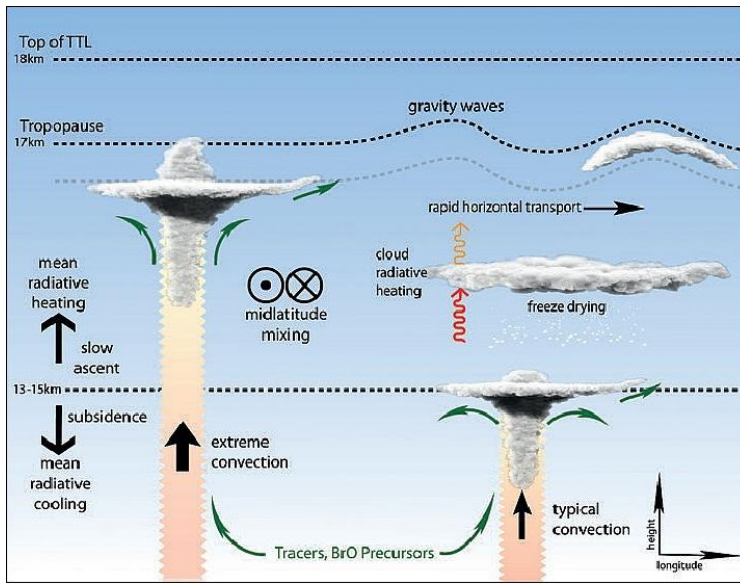
Biases in the ECMWF atmospheric model (2/2)

GPS-RO temperature retrievals provide a global coverage which allows the study of spatial patterns of model error



Temperature first-guess departure with respect to GPS-RO (~70hPa, January 2017)

→ model error is large scale and presents specific features



Strong convection can generate gravity waves that transfer momentum from the troposphere to the stratosphere

This process is not entirely represented in the IFS model and produces cold biases in the lower/middle stratosphere

How to deal with model biases? (weak-constraint 4D-Var)

Unknown forcing is introduced (additive, Gaussian, constant within the assimilation window, no cross-correlation with the background error).

$$\mathbf{x}_k = \mathcal{M}_{k,k-1}(\mathbf{x}_{k-1}) + \eta \quad \text{for} \quad k = 1, \dots, N$$

Cost function depends on the initial state and the model forcing

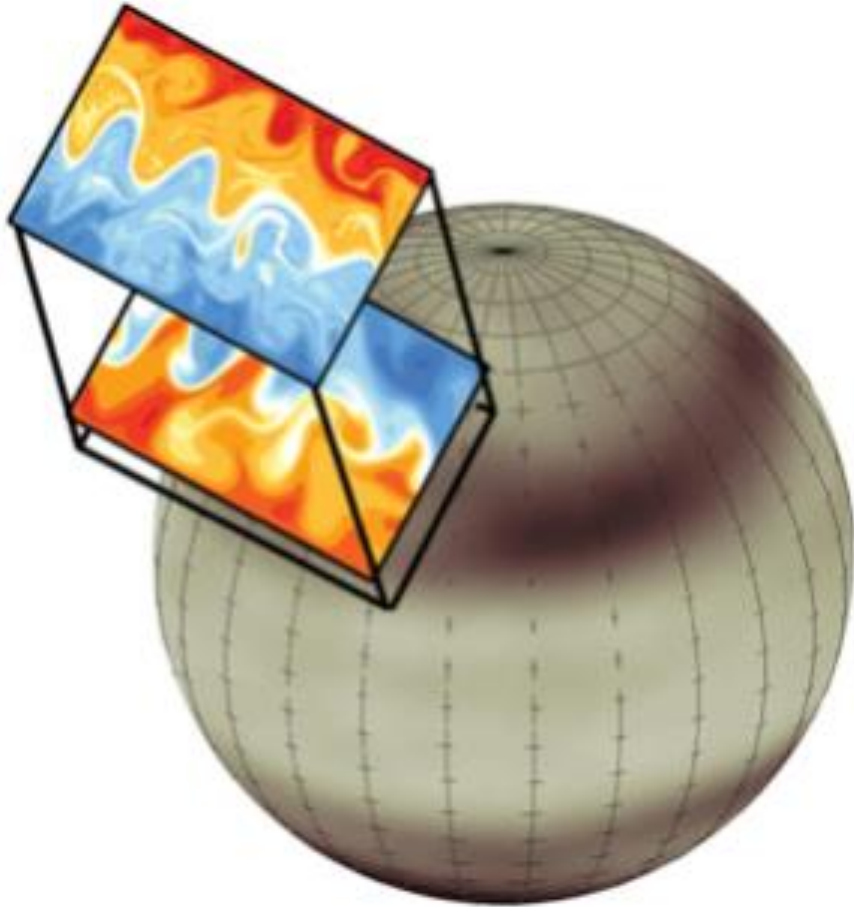
$$\begin{aligned} J_{WC}(\mathbf{x}_0, \eta) &= \frac{1}{2} (\mathbf{x}_0 - \mathbf{x}_0^b)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_0^b) \\ &\quad + \frac{1}{2} \sum_{k=0}^N (\mathcal{H}_k \mathcal{M}_{k,0}(\mathbf{x}_0) - \mathbf{y}_k)^T \mathbf{R}_k^{-1} (\mathcal{H}_k \mathcal{M}_{k,0}(\mathbf{x}_0) - \mathbf{y}_k) \\ &\quad + \frac{1}{2} (\eta - \eta^b)^T \mathbf{Q}^{-1} (\eta - \eta^b) \end{aligned}$$

Set of coupled equations (observations available at time t0 and t1)

$$\mathbf{x}_0 = \mathbf{x}_0^b + \left[\mathbf{B}^{-1} + \sum_{k=0}^1 \mathbf{M}_{k,0}^T \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{M}_{k,0} \right]^{-1} \left(\mathbf{H}_0^T \mathbf{R}_0^{-1} (\mathbf{y}_0 - \mathbf{H}_0 \mathbf{x}_0^b) + \mathbf{M}_{1,0}^T \mathbf{H}_1^T \mathbf{R}_1^{-1} (\mathbf{y}_1 - \mathbf{H}_1 (\mathbf{M}_{1,0} \mathbf{x}_0^b - \eta)) \right)$$

$$\eta = \eta^b + (\mathbf{Q}^{-1} + \mathbf{H}_1^T \mathbf{R}_1 \mathbf{H}_1)^{-1} \mathbf{H}_1^T \mathbf{R}_1^{-1} (\mathbf{y}_1 - \mathbf{H}_1 (\mathbf{M}_{1,0} \mathbf{x}_0 - \eta^b))$$

How weak-constraint 4D-Var works with the QG model?



The Quasi-Geostrophic (QG) model is very important in geophysical fluid dynamics as it describes some aspects of flows in the oceans and atmosphere very well

Experiment framework

→ A bias is introduced in the model

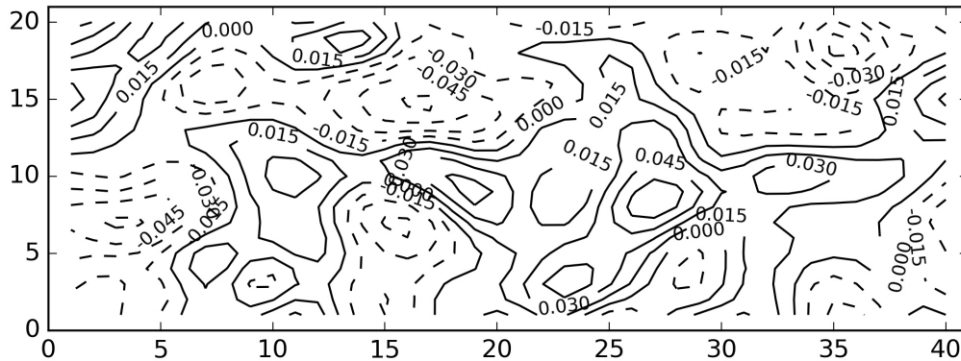
$$x_k = \mathcal{M}_k(x_{k-1}) + \eta \quad \text{for } k = 1, 2, \dots, K$$

→ Observations are generated over 30 days

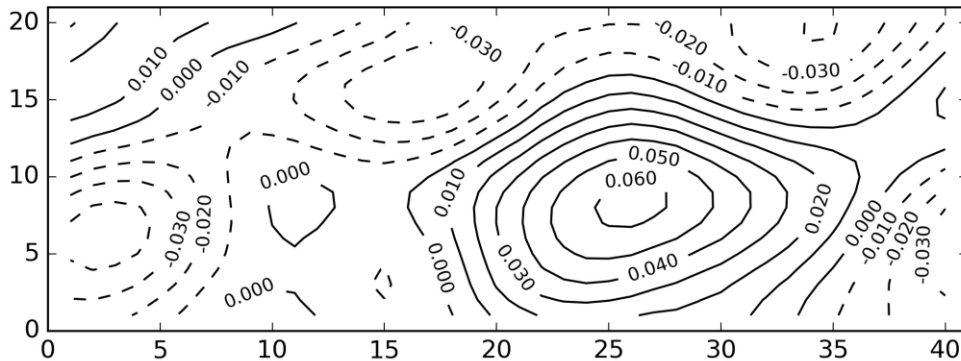
→ Can weak-constraint 4D-Var estimate correctly the model bias?

Covariance matrices in our experiments

Covariance matrix	horizontal correlation	vertical correlation	Scales
B	0.6×10^6	0.2	Short
Q_s	0.6×10^6	0.2	Short
Q_l	1.6×10^6	0.8	Long
R	0.0	0.0	Grid point



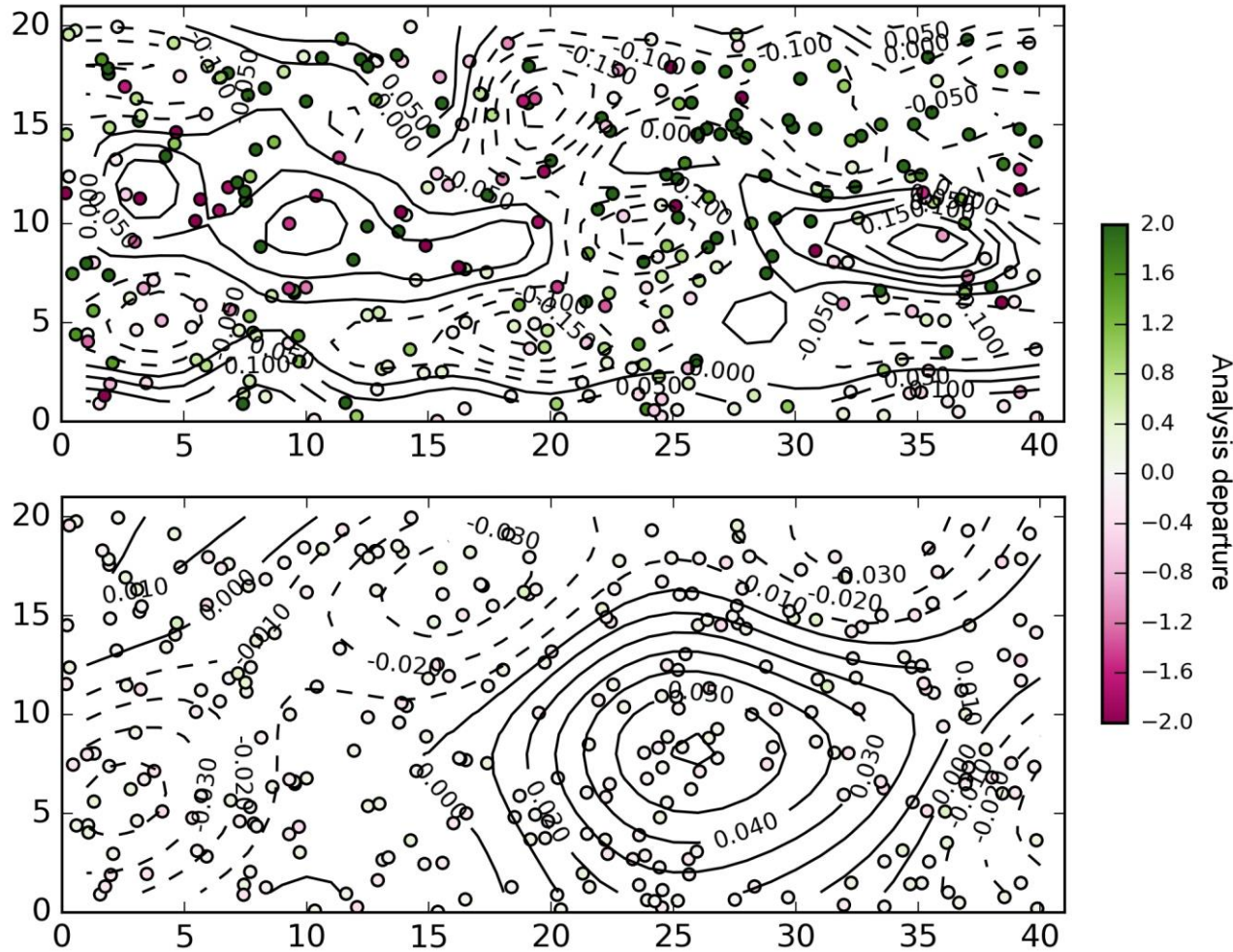
Short scale model bias drawn from Q_s



Long scale model bias drawn from Q_l

Results of weak-constraint 4D-Var (1/3)

Model bias estimate and analysis departure



Short scale model bias

→ poor estimation of the model error and model state

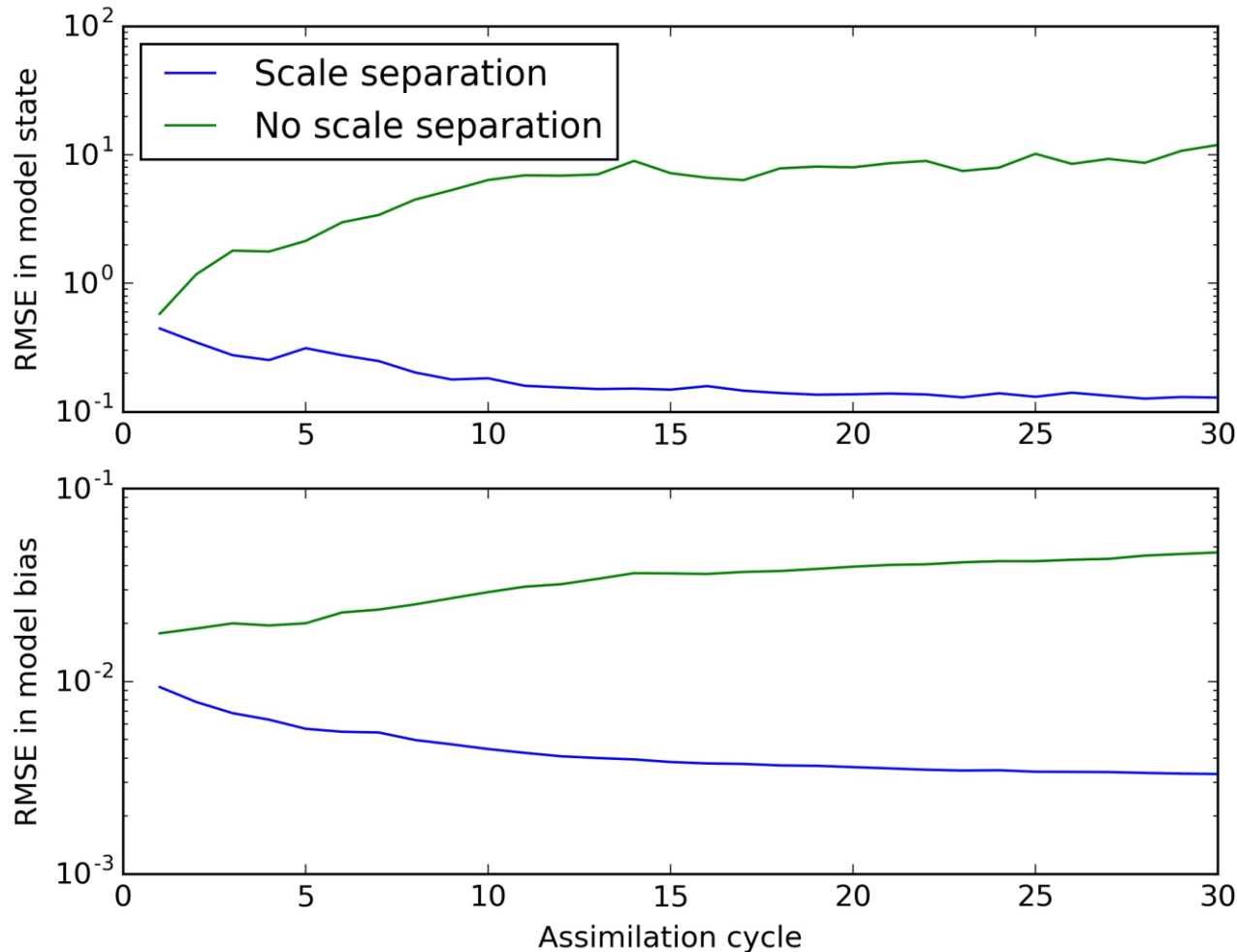
Long scale model bias

→ good estimation of the model error and model state

Results of weak-constraint 4D-Var (2/3)

To get results that are statistically significant,

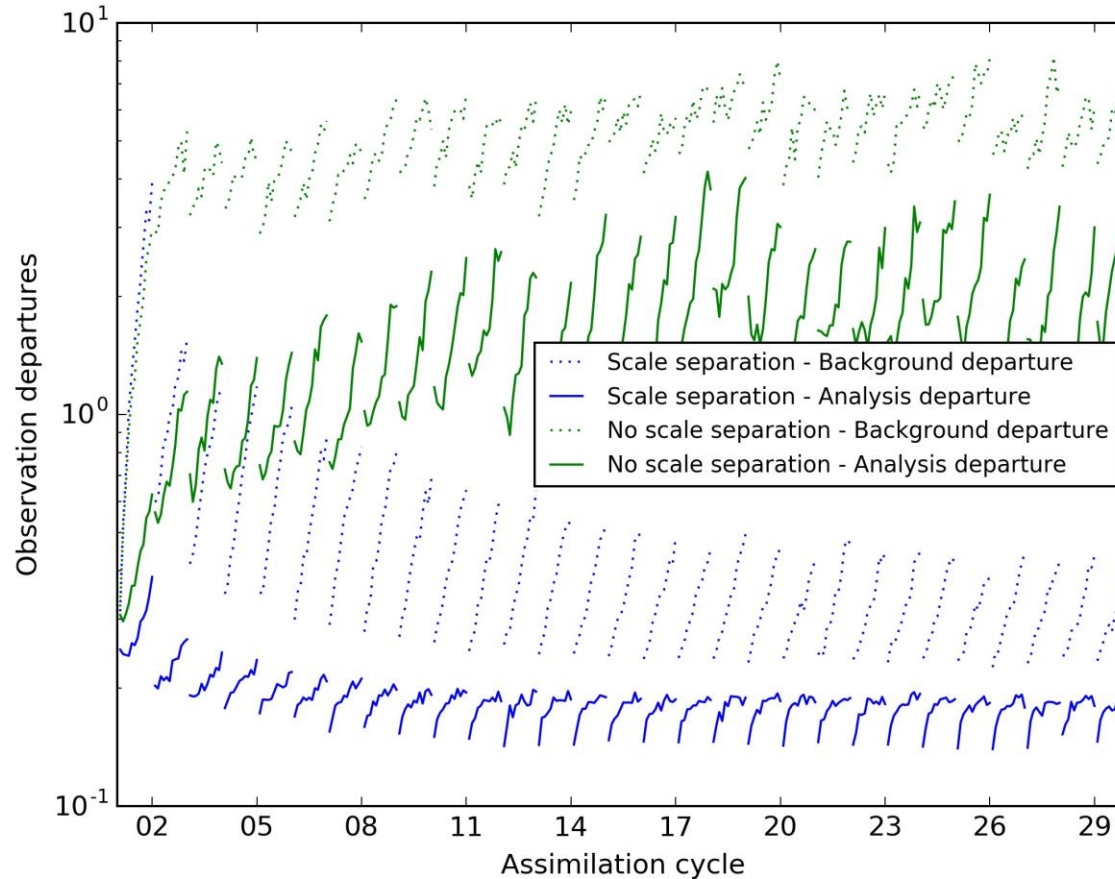
- two experiments are repeated 100 times
- different realisations of the model error drawn from Qs or QI
- compute mean RMSE in model state and model bias



Weak-constraint 4D-Var with scale separation converges towards the true model state and model error

Results of weak-constraint 4D-Var (3/3)

Timeseries of background and analysis departures (averaged over 100 experiments)



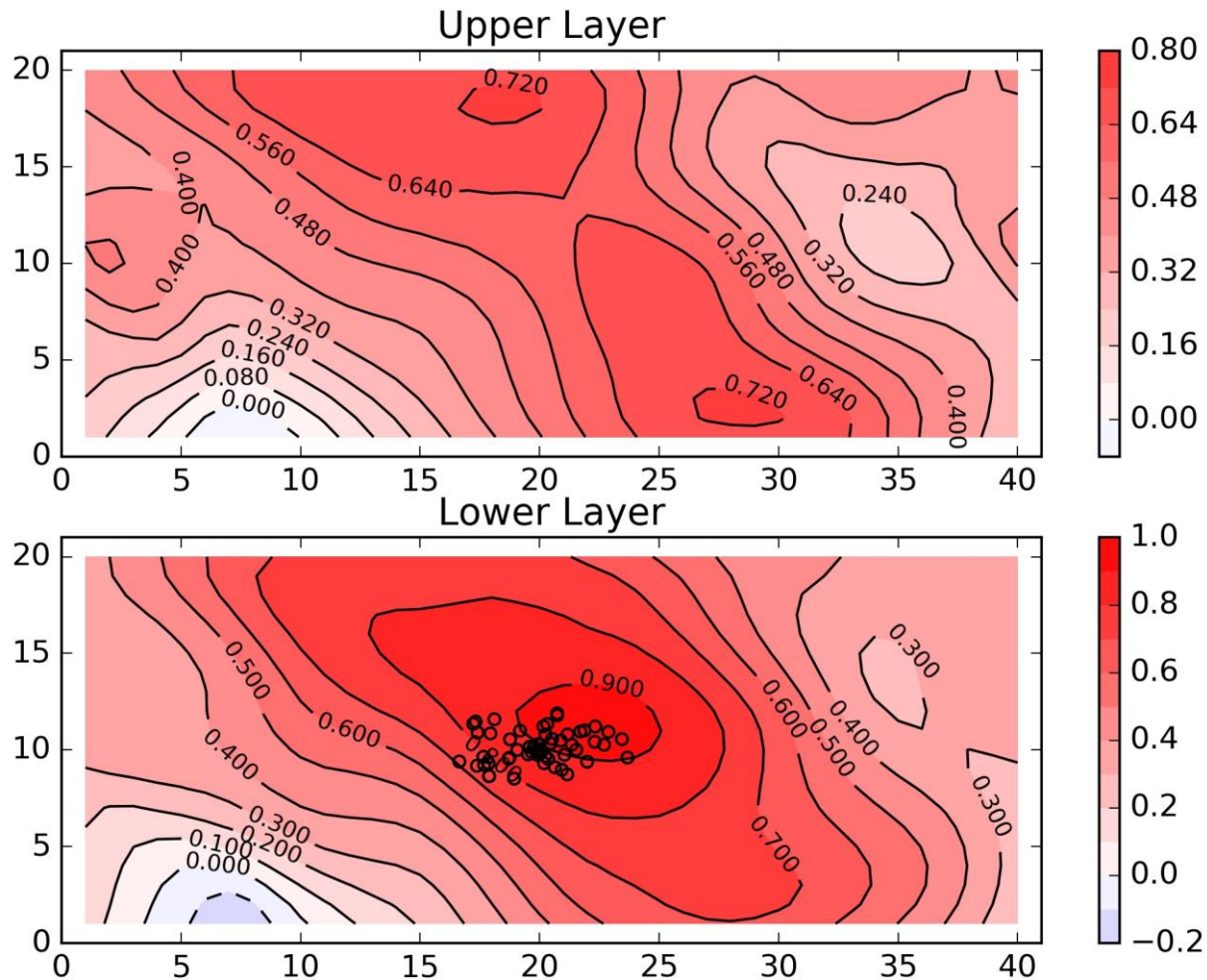
→ only weak-constraint 4D-Var with scale separation fits correctly the observations

→ first-guess trajectory used to compute the background departure is corrected using the model error estimate from the previous assimilation

Impact of the observing system (with scale separation)

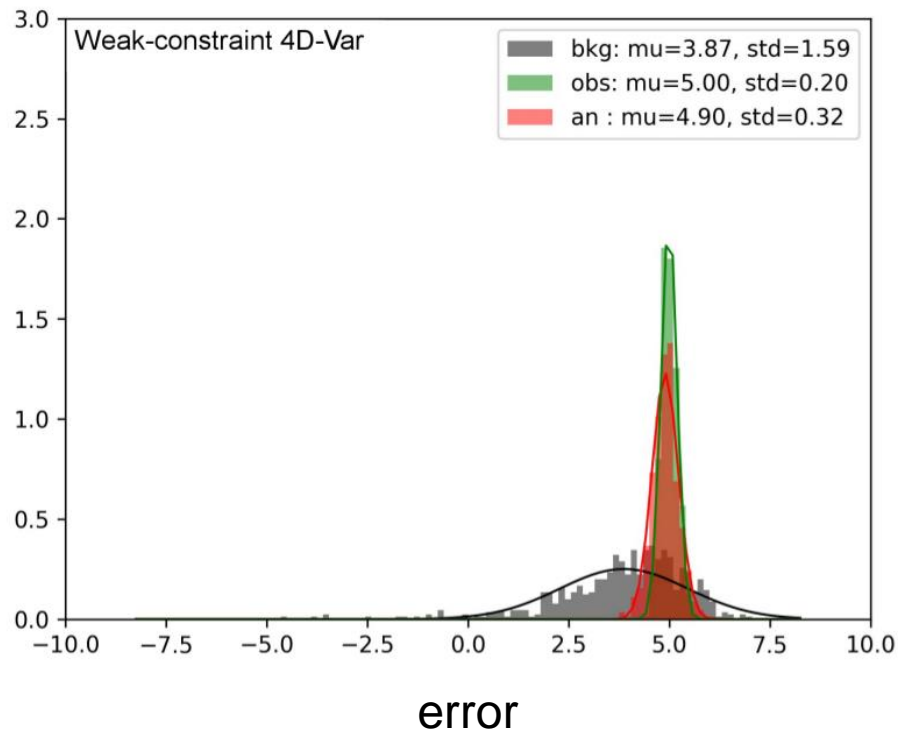
Observing system is changed to mimic the type of dense but localised coverage we would get if an airport was located in the middle of the domain.

Correlation between the model error estimated by weak-constraint 4D-Var with scale separation and the true model error drawn from QI



Impact of observation bias (with scale separation)

A bias is added to the observations



The bias added to create the observations is absorbed in the model error estimation

Understanding the potential and limitations of weak-constraint 4D-Var

P. Laloyaux^{1*}, M. Bonavita¹, M. Chrust¹ and S. Gürol²,

¹ *European Centre for Medium-Range Weather Forecasts, Reading, United Kingdom*

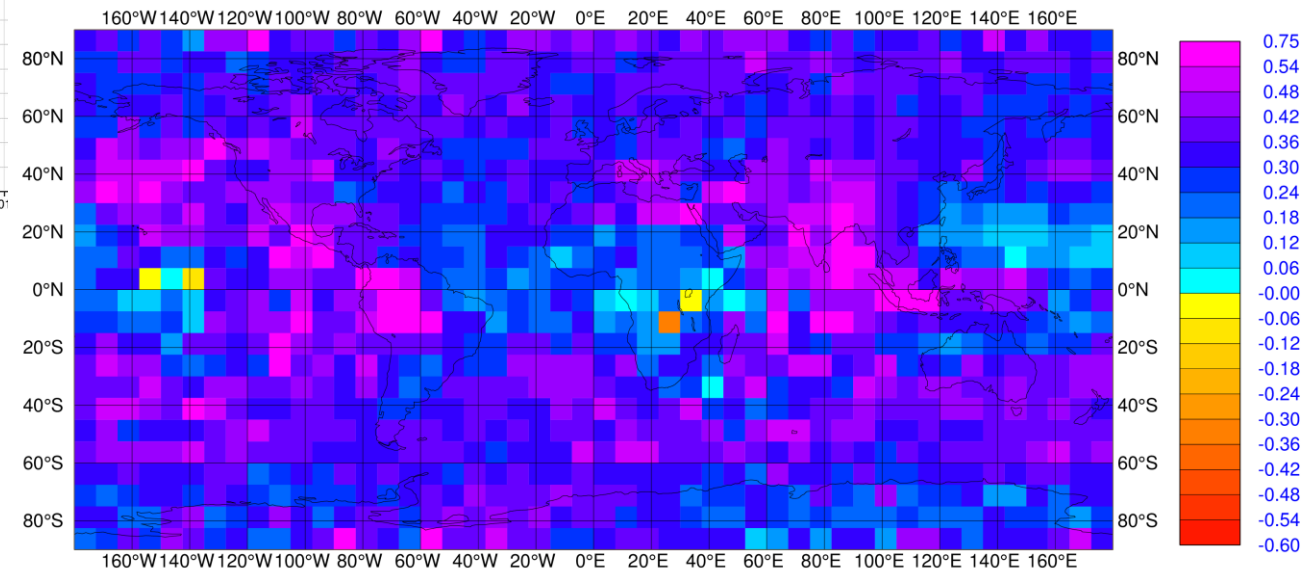
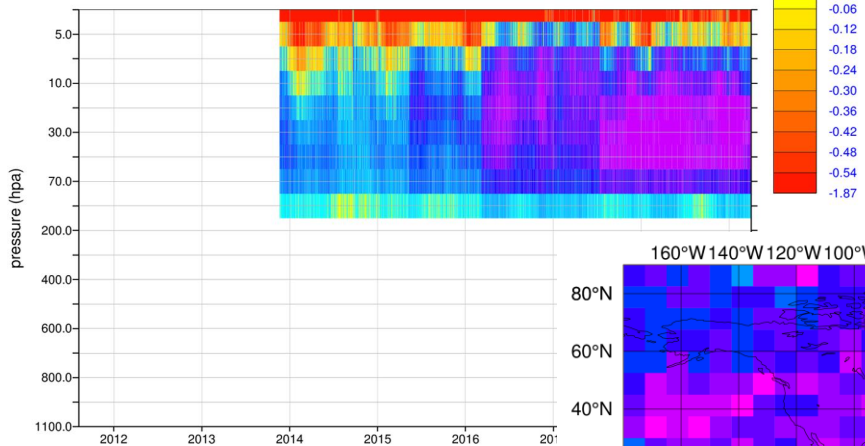
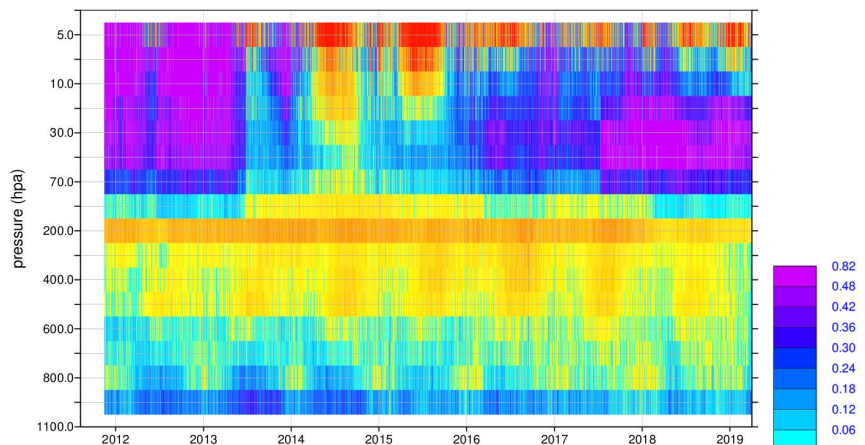
² *Centre Européen de Recherche et de Formation Avancée en Calcul Scientifique, Toulouse, France*

*Correspondence to: P. Laloyaux, European Centre for Medium-Range Weather Forecasts, Shinfield Park, Reading RG2 9AX, United Kingdom, Email: patrick.laloyaux@ecmwf.int

WC4DVAR can accurately estimate the model errors and the initial state only when

- background and model errors have different spatial scales
- the observing system is spatially homogeneous
- the observing system is unbiased

Biases in the ECMWF atmospheric model

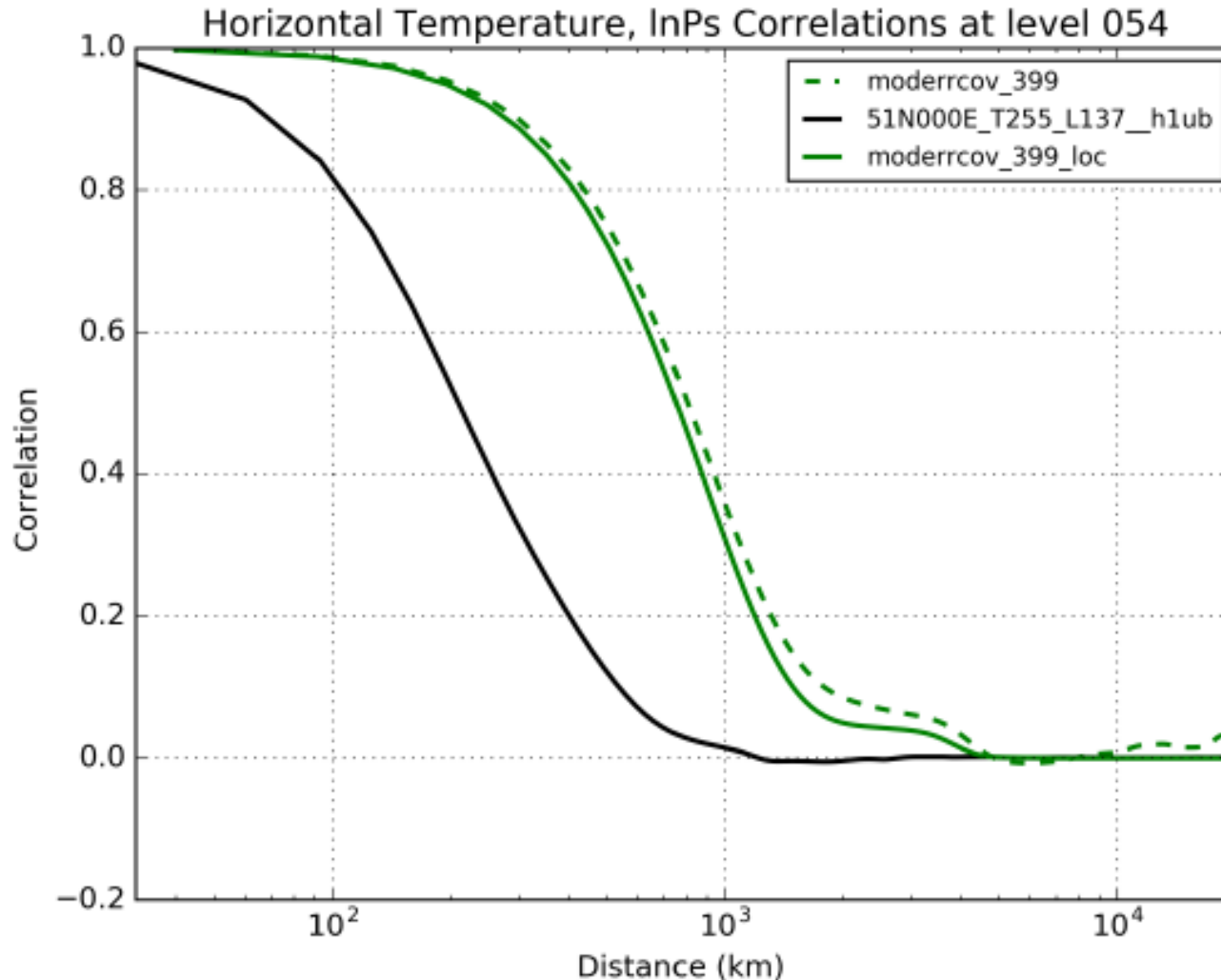


Weak-constraint 4D-Var with scale separation in IFS

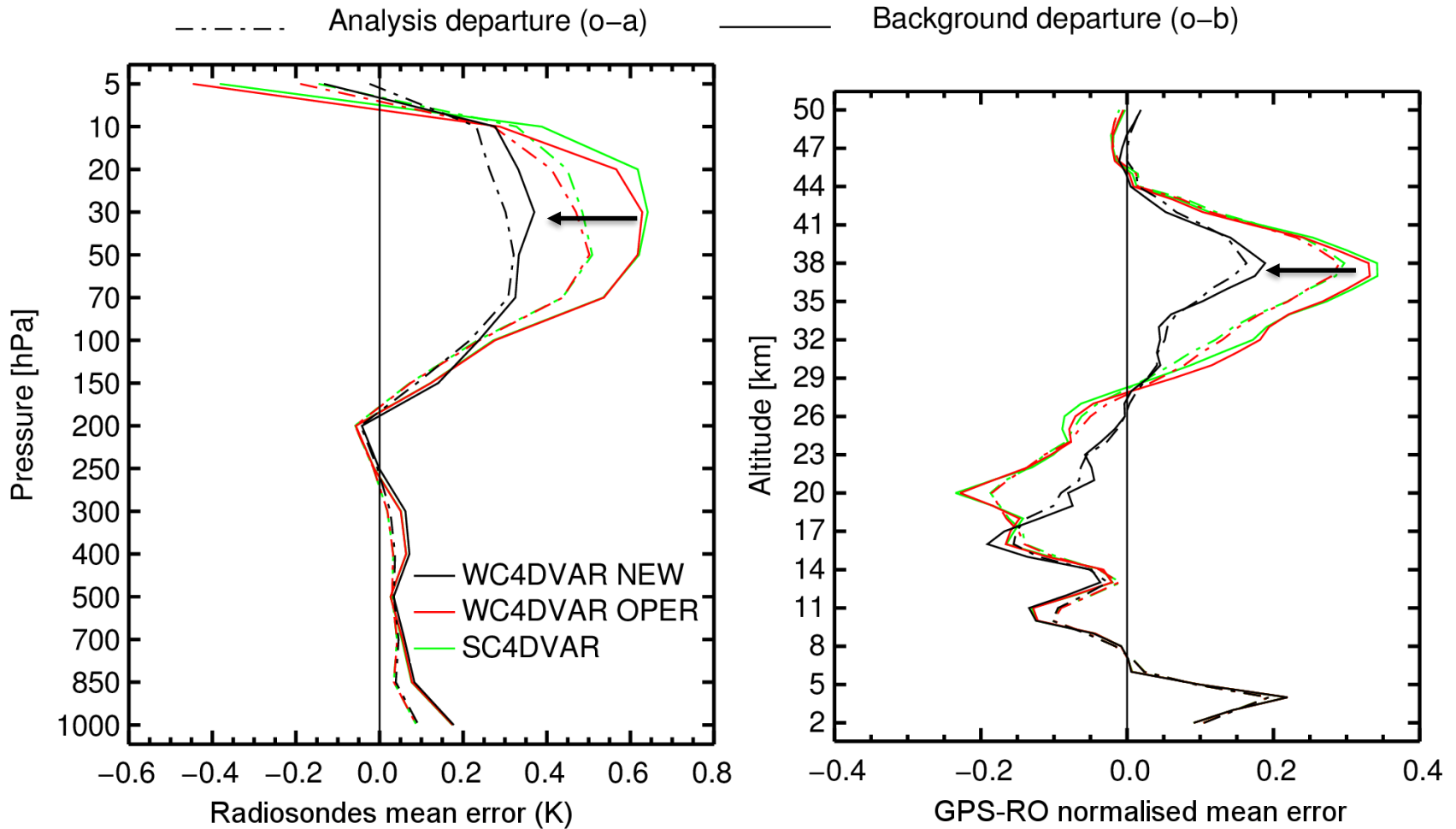
Horizontal correlation in the covariance matrices

→ **B** corrects the small scales

→ **Q** corrects the large scale

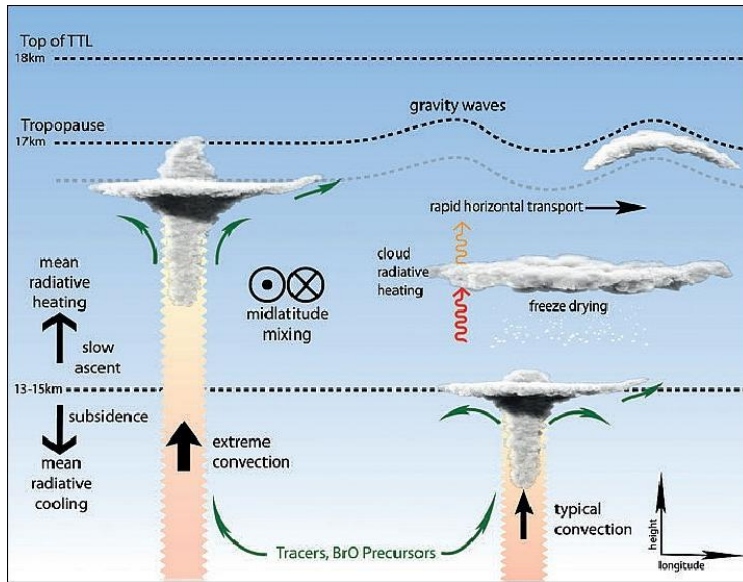


Weak-constraint 4D-Var with scale separation in IFS (1/3)

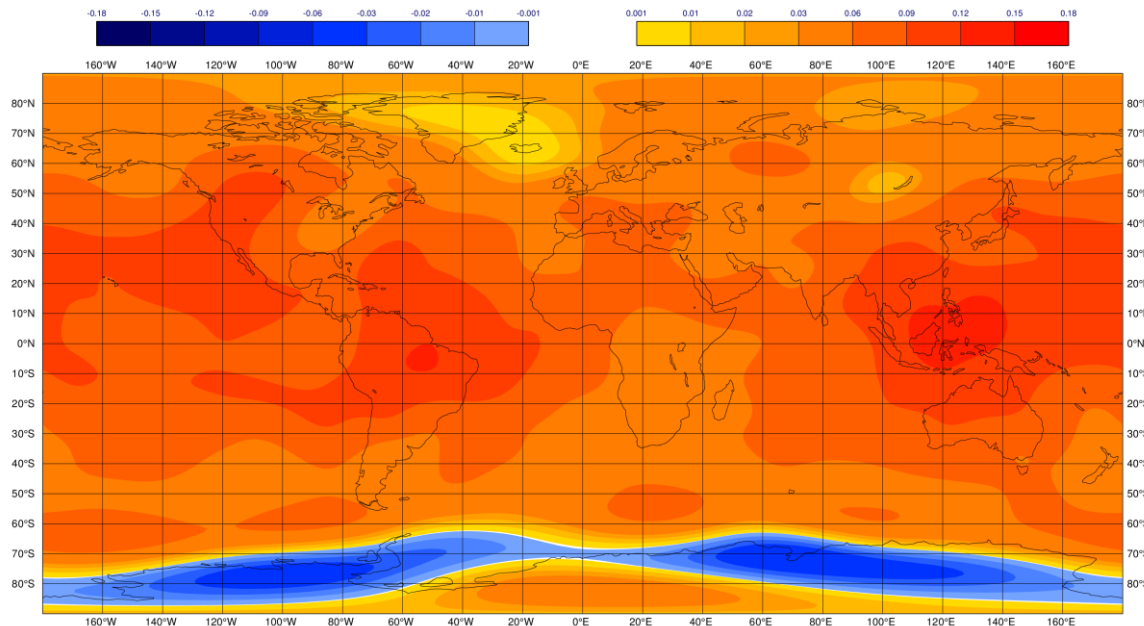


Temperature bias is reduced up to 50% with respect to radiosondes and GPS-RO

Weak-constraint 4D-Var with scale separation in IFS (2/3)



Cold biases in the lower/middle stratosphere over strong convective regions



Model error forcing from weak-constraint 4D-Var at 70 hPa

→ correcting the bias from the missing gravity waves

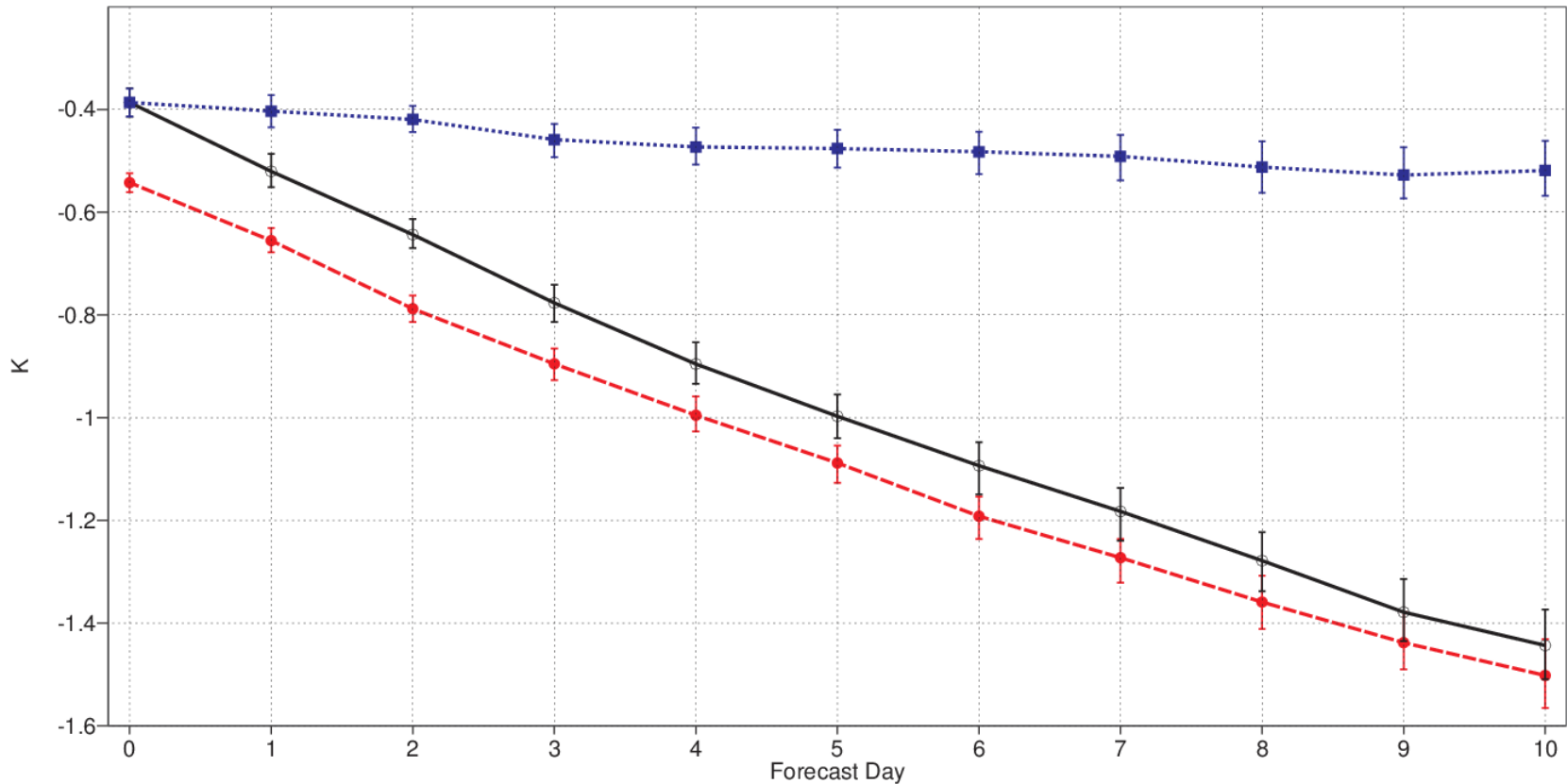
Weak-constraint 4D-Var with scale separation in IFS (3/3)

Mean forecast error with respect to radiosondes (70hPa)

Strong-constraint 4D-Var

Weak-constraint 4D-Var (analysis only)

Weak-constraint 4D-Var (corrected forecast)



Understanding the potential and limitations of weak-constraint 4D-Var

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WC4DVAR can accurately estimate the model errors and the initial state only when

- background and model errors have different spatial scales
- the observing system is spatially homogeneous
- the observing system is unbiased

WC4DVAR with scale separation will be implemented in CY47R1

Observation bias correction (VarBC) will be reviewed to attribute errors to their proper sources