PSO Algorithm for Optimum Well Placement subject to Realistic Field Development Constraints

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Well Placement Problem

- Common formulation of well placement problem:

$$\max_{\boldsymbol{\zeta},\mathbf{u}^n} [J = \sum_{n=0}^{N-1} L^n(\mathbf{x}^{n+1}, \boldsymbol{\zeta}, \mathbf{u}^n)],$$

subject to:

$$\boldsymbol{\zeta}^d \leq \boldsymbol{\zeta} \leq \boldsymbol{\zeta}^u,$$

$$\mathbf{u}^d \leq \mathbf{u}^n \leq \mathbf{u}^u,$$

$$\mathbf{x}^0 = \mathbf{x}_0,$$

$$g^n(\mathbf{x}^{n+1}, \mathbf{x}^n, \boldsymbol{\zeta}, \mathbf{u}^n) = 0, \; n = 0, 1, \cdots, N-1.$$

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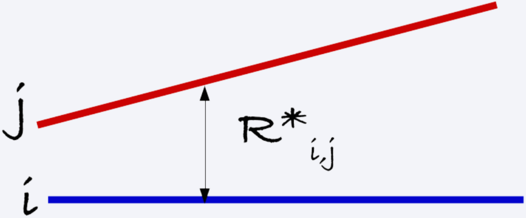
Motivation

- Problem: Engineering experiences are not included.
- Valuable solution depends on
  - Identification of limitations,
  - Translation of them into constraints.
- The success of the optimization effort relies on
  - Efficient search algorithm,
  - Constraint-handling techniques.

Well Placement Constraints

- Well distance $C_{wd} : R^*_{i,j} \geq d_{min}$
- Well length $C_{wl} : L_i = \|\zeta_i^h - \zeta_i^t\|_2,\ l^i_{min} \leq L_i \leq l^i_{max}$
- Reservoir bound $C_{rb} : \zeta_i^h \in R_i^h, \quad \zeta_i^t \in R_i^t$
- Well orientation
  $C_{wo} : \theta_{i,j} = \arccos\left|\frac{(\zeta_i^h-\zeta_i^t)\cdot(\zeta_j^h-\zeta_j^t)}{\|\zeta_i^h-\zeta_i^t\|_2\|\zeta_j^h-\zeta_j^t\|_2}\right| \leq \theta_{max}$



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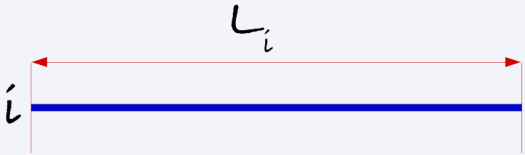

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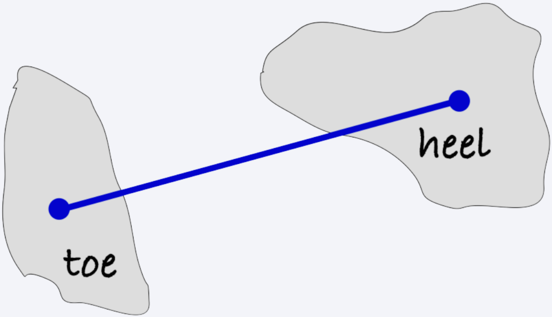

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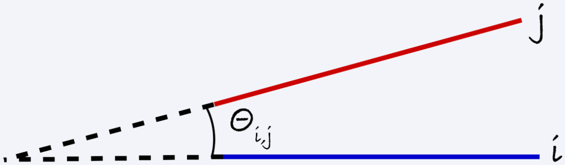


General Form of Well Placement Problem

$$\min -\text{NPV},$$

subject to:

$$C_i(\boldsymbol{\zeta}) \geq 0, \quad i \in \{wd, wl, rb, wo\},$$

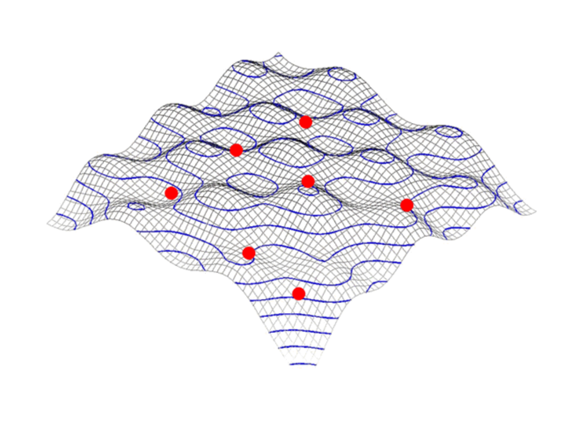
$$\mathbf{u}^d \leq \mathbf{u}^n \leq \mathbf{u}^u,$$

$$\mathbf{x}^0 = \mathbf{x}_0,$$

$$g^n(\mathbf{x}^{n+1}, \mathbf{x}^n, \boldsymbol{\zeta}, \mathbf{u}^n) = 0, \; n = 0, 1, \cdots, N-1.$$

Particle Swarm Optimization (PSO)

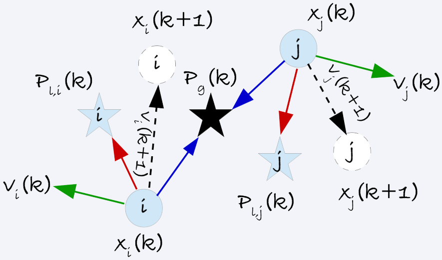
- PSO provides comparable or better results than binary GA (Onwunalu and Durlofsky, 2010).



Particle Swarm Optimization (PSO)

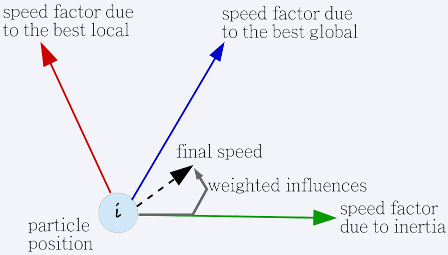
$$\boldsymbol{\nu}_i(k+1) = \boldsymbol{\nu}_i(k) + c_1\rho_1(k)(\mathbf{p}_{l,i}(k) - \mathbf{x}_i(k)) + c_2\rho_2(k)(\mathbf{p}_{g,i}(k) - \mathbf{x}_i(k)),$$

$$\mathbf{x}_i(k+1) = \mathbf{x}_i(k) + \boldsymbol{\nu}_i(k+1).$$



Inertia Weight

$$\hat{\boldsymbol{\nu}}_i(k+1) = w(k)\boldsymbol{\nu}_i(k) + c_1\rho_1(k)(\mathbf{p}_{l,i}(k) - \mathbf{x}_i(k)),$$
$$+ c_2\rho_2(k)(\mathbf{p}_{g,i}(k) - \mathbf{x}_i(k)),$$
$$\nu_i^j(k+1) = \mathsf{sign}(\hat{\nu}_i^j(k+1)) \min\{|\hat{\nu}_i^j(k+1)|, \nu_{max}^j\},$$
$$\mathbf{x}_i(k+1) = \mathbf{x}_i(k) + \boldsymbol{\nu}_i(k+1),$$
$$\nu_{max}^j = \lambda(u^j - l^j), \quad w(k) = w_0 - \frac{k}{K}(w_0 - w_1).$$



Method 1: Penalty function

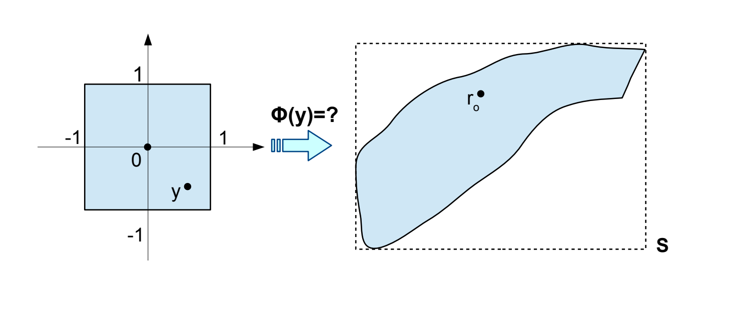
- Merit function

$$\phi_1(\zeta, \mu) = -(\mathsf{NPV})_{sc} + \mu \sum_i \max\{0, -(C_i)_{sc}\},$$

- Penalty parameter (μ) grows with iteration number.

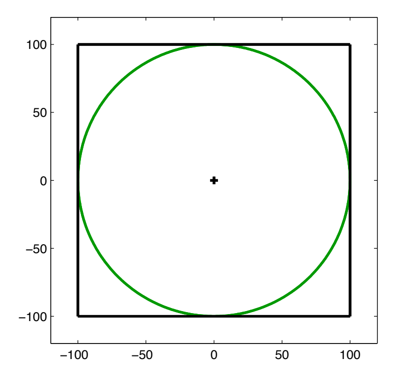
Method 2: Decoder

- A homomorphous mapping between an n-dimensional cube and a feasible search space (Koziel and Michalewicz, 1999).



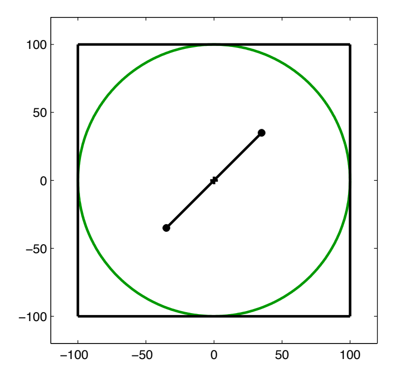
Introducing Decoder for Placing one Horizontal Well

- Constraints: Both toe and heel should stay in the circle (feasible region),
- Variables: Cartesian coordinate for both heel (x_h, y_h) and toe (x_t, y_t)



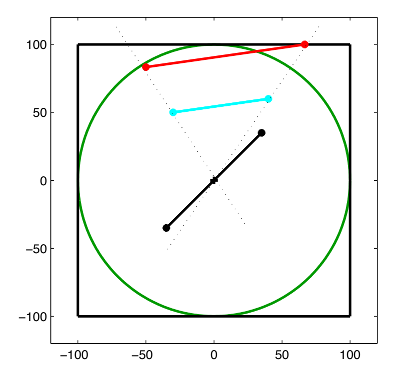
Introducing Decoder for Placing one Horizontal Well

- Step 1: Define reference $r_0 = \begin{bmatrix} 35 & 35 & -35 & -35 \end{bmatrix}$
- Step 2: The input of decoder should stay in the cube $[-1, 1]^4$ $y = \begin{bmatrix} 0.4 & 0.6 & -0.3 & 0.5 \end{bmatrix}$



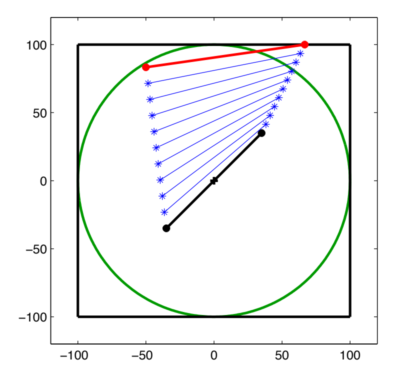
Introducing Decoder for Placing one Horizontal Well

- Step 3: Calculate
  $y/y_{max} = \frac{1}{0.6}\begin{bmatrix}0.4 & 0.6 & -0.3 & 0.5\end{bmatrix}$
- Step 4: Map $g(y)$ to s
  $s = g(y/y_{max}) = \begin{bmatrix}66.7 & 100 & -50 & 83.3\end{bmatrix}$
  $g(y) = (y - \frac{(u-l)}{2}) + \frac{u+l}{2}$



Introducing Decoder for Placing one Horizontal Well

- Step 5: Define line segment between s and $\mathbf{r_0}$: $L(r_0, s) = r_0 + t(s - r_0)$
- Step 6: Find t_0 where L intersects the boundary of circle: $t_0 = 0.72$



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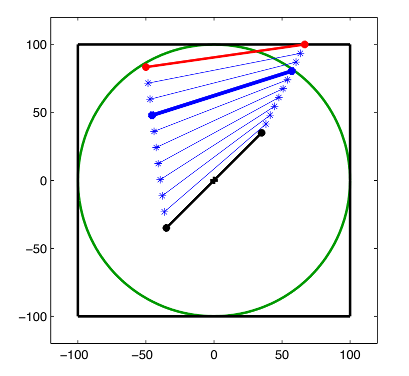
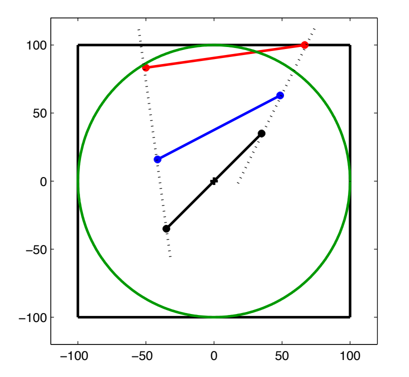


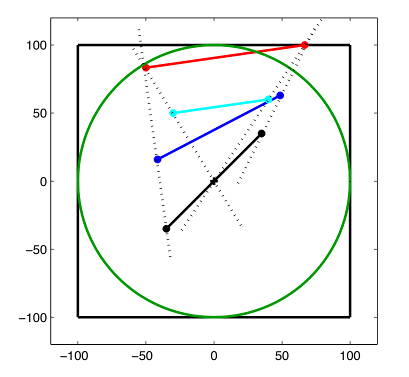
Introducing Decoder for Placing one Horizontal Well

- Step 7: Calculate $\phi(y)$:
  $\phi(y) = r_0 + y_{max} t_0 (s - r_0)$



Introducing Decoder for Placing one Horizontal Well

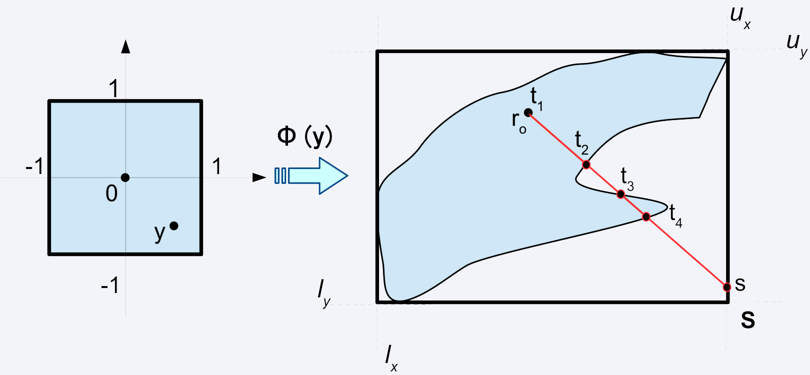
- $g(y)$
- $g(y/y_{max})$
- $r_0 + y_{max}t_0(s - r_0)$



Additional Constraints and Non-Convex Feasible Set

- Non-convex feasible set if:
  - Non-convex feasible region,
  - Include other constraints.
- In the case of non-convex feasible set:
  - All steps are same,
  - Several feasible interval: $[t_1, t_2], \cdots [t_{2k-1}, t_{2k}]$
  - Define new map: $\gamma : (0, 1] \to \cup_{i=1}^{k} (t_{2i-1}, t_{2i}]$

Non-Convex Feasible Space



$$\gamma : (0,1] \rightarrow \cup_{i=1}^{k} (t_{2i-1}, t_{2i}]$$

General Form of Decoder

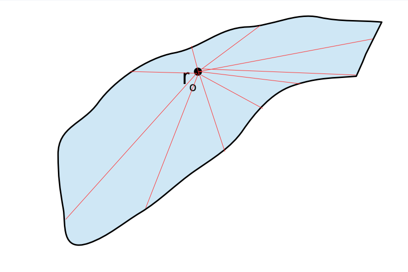
$$\phi(\mathbf{y}) = \begin{cases} \mathbf{r}_o + t_o \cdot (g(\mathbf{y}/y_{max} - \mathbf{r}_o)) & \text{if } \mathbf{y} \neq \mathbf{0} \\ \mathbf{r}_o & \text{if } \mathbf{y} = \mathbf{0} \end{cases}$$

$$y_{max} = \max_{i=1}^{n} \mid y_i \mid,$$

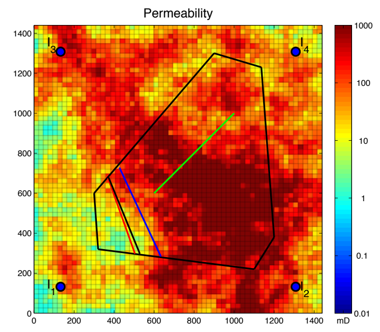
$$t_0 = \gamma(\mid y_{max} \mid).$$

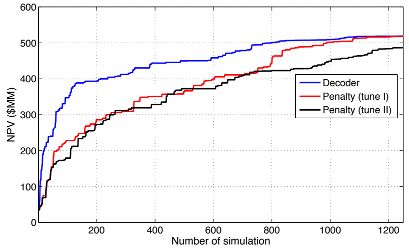
Decoder

- There is no need for any additional parameters,
- Always return a feasible solution,
- The map has locality feature, if any line segment, originates from the reference point, intersect the feasible search space just at one point.



Case Study I

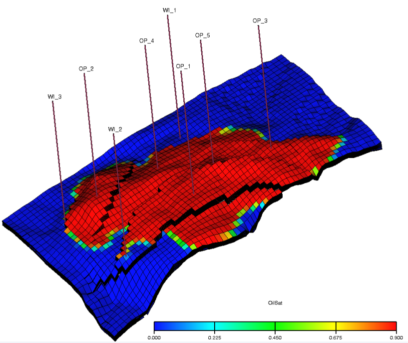


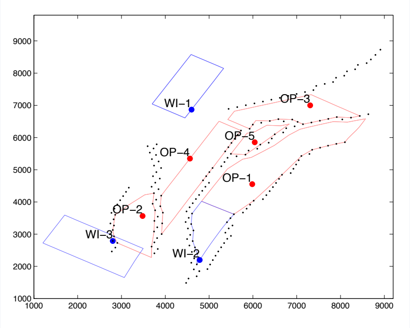


Algorithm	Best $(\times 10^8)$	Mean $(\times 10^8)$	Relative standard deviation (%)
Decoder	5.28	5.19	2.8
Penalty(tune I)	5.26	5.17	2.7
Penalty(tune II)	5.24	4.86	6.8

Case Study II: Regions Setting for Decoder

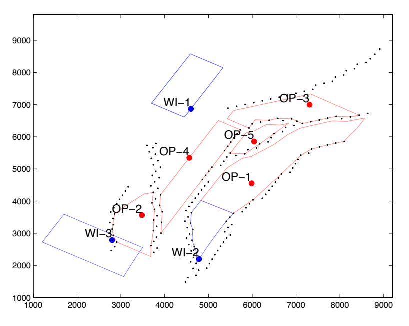
- 5 producers and 3 injectors,
- one realization,
- fixed production settings,
- $40 \times 64 \times 14 = 35,840$ grid cells.



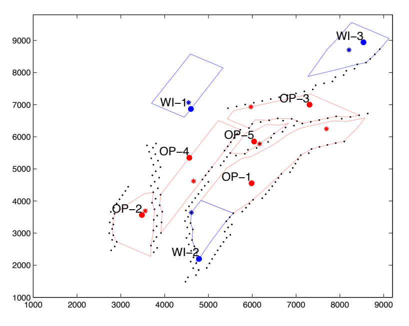


Case Study II: Regions Setting for Decoder

Initial search regions

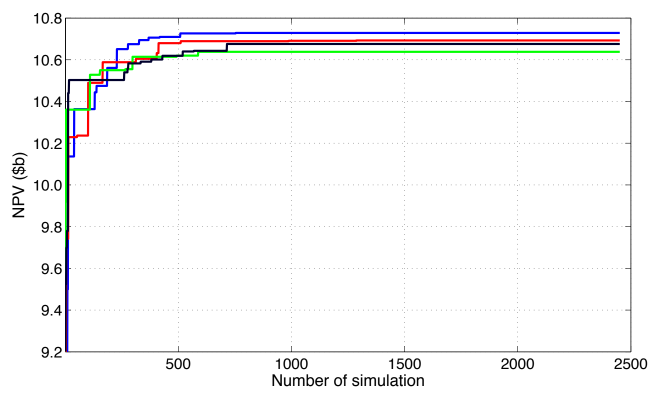


Improved search regions



Case Study II: results

$$n_p = 49, \; n_g = 50$$



Conclusion and Future Work

- Conclusion:
    - Improve the decision-making support by introducing realistic well placement constraints,
    - Couple decoder with the PSO algorithm,
    - Compare to the penalty method, the decoder can be used efficiently.
- Future work:
    - Applying this methodology to more complex cases,
    - Geological uncertainty,
    - Variable production strategy.

Thank You!

Koziel, S., Michalewicz, Z., Mar. 1999. Evolutionary algorithms, homomorphous mappings, and constrained parameter optimization. Evol. Comput. 7 (1), 19–44. URL `http://dx.doi.org/10.1162/evco.1999.7.1.19`

Onwunalu, J., Durlofsky, L., 2010. Application of a particle swarm optimization algorithm for determining optimum well location and type. Comput. Geosci. 14 (1), 183–198.