Observability-Based Ensemble Initiation for the EnKF in History Matching Problems

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EnKF Workshop
June 1 | Balestrand, Norway

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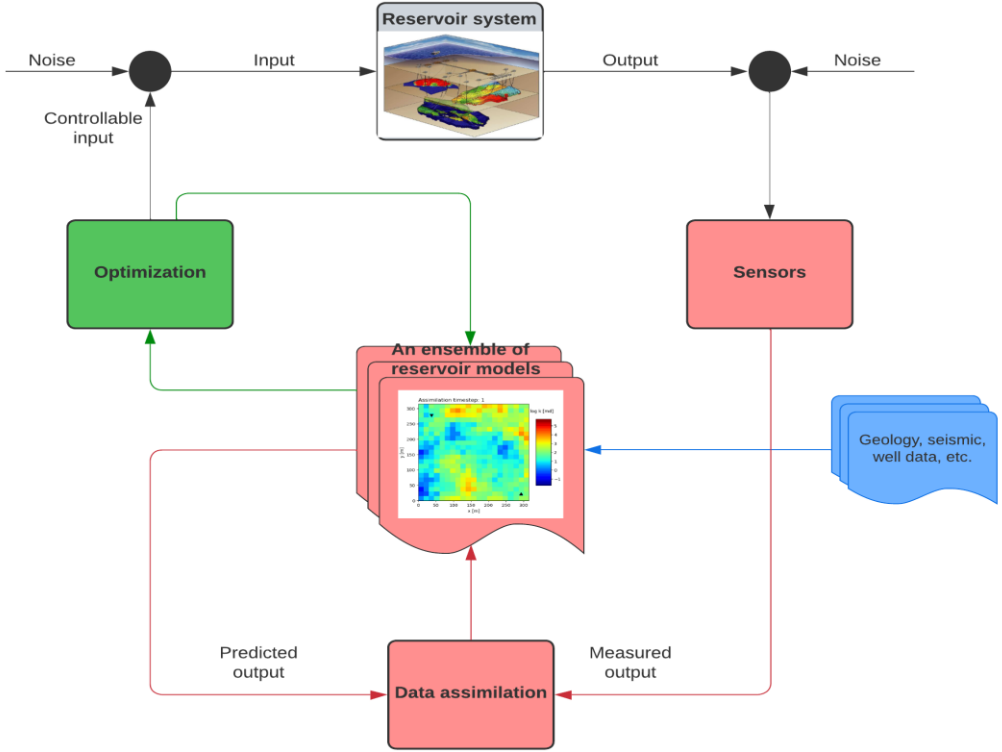
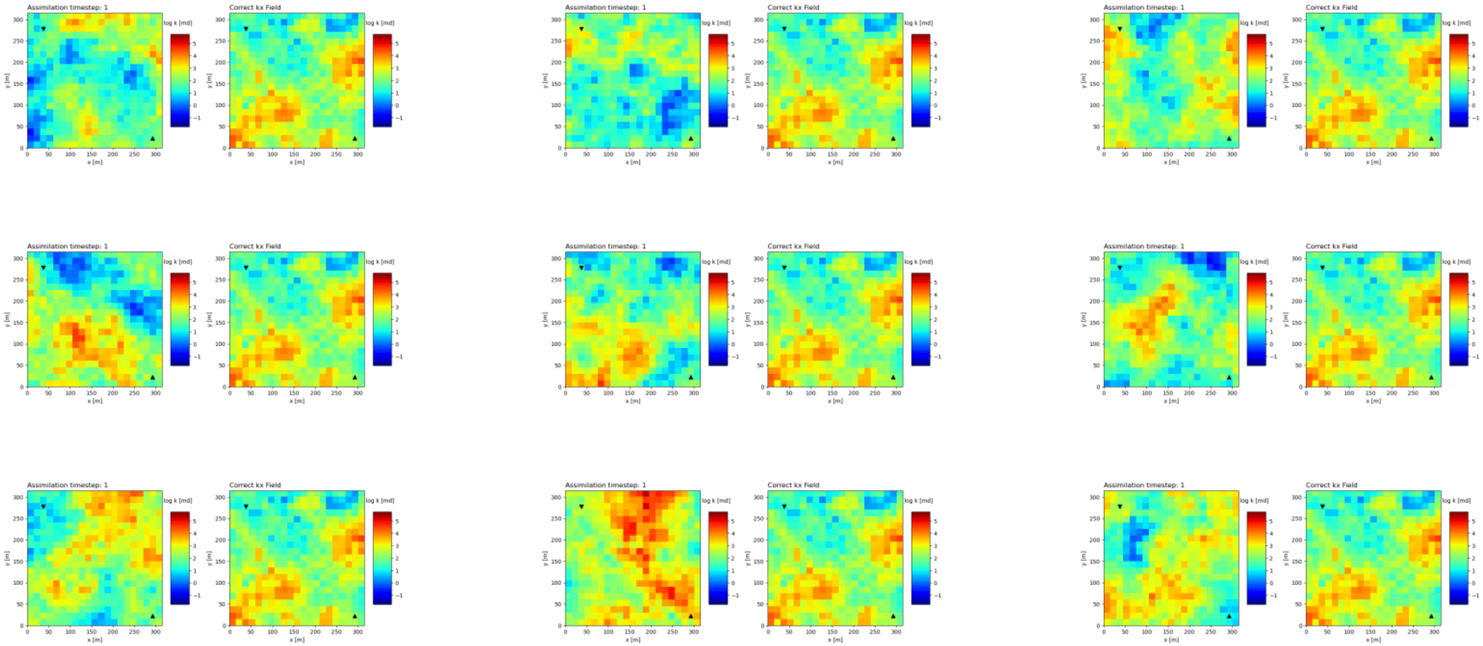
Motivation

- Ensemble initiation is essential for the EnKF performance.
- Improvements can lead to significant economical benefits.
- Generating the initial ensemble in the high-observable directions.

Closed Loop Reservoir Management (CLRM)



2D Reservoir Model Update using EnKF



Observability-Aware EnKF: Main idea

- Sampling in directions that are strongly observable from the measured outputs.

- **Two advantages**: 1. Perturbations are orthogonal (reduces redundancy)
  2. In the high-observable directions.

- **Challenges**: 1. Observability analysis in multiphase heterogenous reservoirs.
  2. Computing sensitivities (with respect to states and parameters).

Problem Assumptions

- Model dynamics: $g(x_k, x_{k+1}, u_k) = 0$
- Observation model: $y_k = f(x_k, u_k)$
- States: $x = [S_w^T \ p_o^T]^T$
- Inputs: $u = [q_{inj} \ p_{BHP_{Prod}}]$
- Outputs: $y = [p_{BHP_{inj}} \ q_{w_{prod}} \ q_{o_{prod}}]$
- Initial conditions: $x_0 = [S_{w_0}^T \ p_{o_0}^T]^T$

Sensitivity calculations

- Balance equation (fully-implicit simulator)

$$f\left(x_{(i)}, x_k, u_k, m\right) = r_{(i)}$$
$$f\left(x_{k+1}, x_k, u_k, m\right) = 0$$

- Total derivatives

$$\delta r = 0 \Rightarrow \delta x_{k+1} = -M_1^{-1} M_2 \delta x_k - M_1^{-1} M_3 \delta u_k - M_1^{-1} M_4 \delta m$$

- Define a state-space model

$$\tilde{x}_k = \begin{Bmatrix} x_k \\ m \end{Bmatrix} \in R^{N_x + N_m}$$

$$\tilde{x}_{k+1} = A \tilde{x}_k + B u_k$$

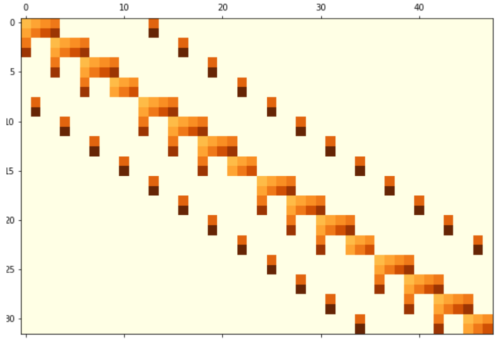
$$A = \begin{bmatrix} -M_1^{-1} M_2 & -M_1^{-1} M_4 \\ 0 & I \end{bmatrix}$$

$$B = \begin{bmatrix} -M_1^{-1} M_3 \\ 0 \end{bmatrix}$$

Sensitivity calculations: continue

$$M_1 = \begin{bmatrix} \frac{\partial r_{w,0}}{\partial s_{w,0}} & \frac{\partial r_{w,0}}{\partial p_{o,0}} & \frac{\partial r_{w,0}}{\partial s_{w,1}} & \frac{\partial r_{w,0}}{\partial p_{o,1}} & \cdots & \frac{\partial r_{w,0}}{\partial s_{w,n_g}} & \frac{\partial r_{w,0}}{\partial p_{o,n_g}} \\ \frac{\partial r_{o,0}}{\partial s_{w,0}} & \frac{\partial r_{o,0}}{\partial p_{o,0}} & \frac{\partial r_{o,0}}{\partial s_{w,1}} & \frac{\partial r_{o,0}}{\partial p_{o,1}} & \cdots & \frac{\partial r_{o,0}}{\partial s_{w,n_g}} & \frac{\partial r_{o,0}}{\partial p_{o,n_g}} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial r_{w,n_g}}{\partial s_{w,0}} & \frac{\partial r_{w,n_g}}{\partial p_{o,0}} & \frac{\partial r_{w,n_g}}{\partial s_{w,1}} & \frac{\partial r_{w,n_g}}{\partial p_{o,1}} & \cdots & \frac{\partial r_{w,n_g}}{\partial s_{w,n_g}} & \frac{\partial r_{w,n_g}}{\partial p_{o,n_g}} \\ \frac{\partial r_{o,n_g}}{\partial s_{w,0}} & \frac{\partial r_{o,n_g}}{\partial p_{o,0}} & \frac{\partial r_{o,n_g}}{\partial s_{w,1}} & \frac{\partial r_{o,n_g}}{\partial p_{o,1}} & \cdots & \frac{\partial r_{o,n_g}}{\partial s_{w,n_g}} & \frac{\partial r_{o,n_g}}{\partial p_{o,n_g}} \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial r_{w,bw_{inj1}}}{\partial q_{inj,bw_{inj1}}} & 0 & 0 & 0 & 0 \\ \frac{\partial r_{o,bw_{inj1}}}{\partial q_{inj,bw_{inj1}}} & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \frac{\partial r_{w,bw_{prodn_{prod}}}}{\partial p_{prod,bw_{prodn_{prod}}}} \\ 0 & 0 & 0 & 0 & \frac{\partial r_{o,bw_{prodn_{prod}}}}{\partial p_{prod,bw_{prodn_{prod}}}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



Sparsity pattern example of M4 for 4 x 4 model

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Sensitivity calculations: continue

- The same for the output equation

$$y_k(x_k, u_k, m) = 0.$$

$$y_k = C\tilde{x}_k + Du_k$$

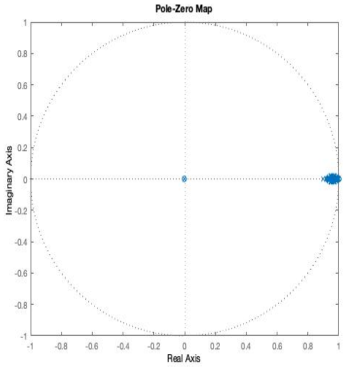
- Intrusive computation of derivatives using algorithmic differentation

$$C = \begin{bmatrix} N_1 & N_3 \end{bmatrix}$$

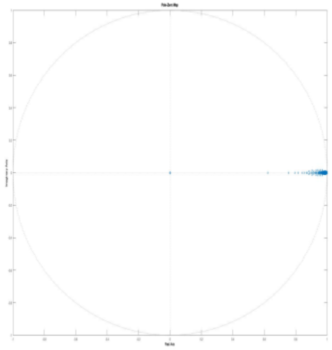
$$D = N_2$$

$$N_1 = \begin{bmatrix} 0 & \cdots & \frac{\partial p_{inj,b_{w_{inj1}}}}{\partial s_{w,b_{w_{inj1}}}} & \frac{\partial p_{inj,b_{w_{inj1}}}}{\partial p_{o,b_{w_{inj1}}}} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & \frac{\partial q_{o,b_{w_{prodn_{prod}}}}}{\partial s_{w,b_{w_{prodn_{prod}}}}} & \frac{\partial q_{o,b_{w_{prodn_{prod}}}}}{\partial p_{o,b_{w_{prodn_{prod}}}}} & \cdots & 0 \end{bmatrix}$$

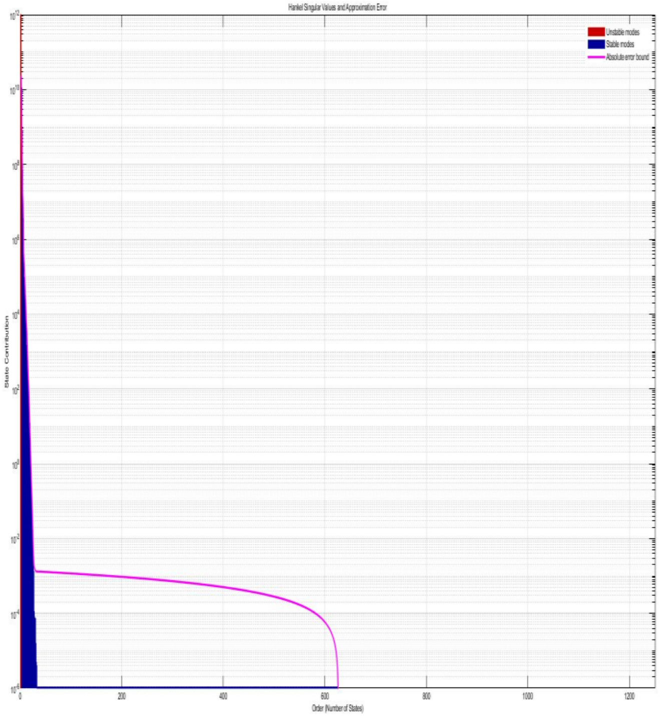
System Analysis: Poles/Zeros and HSV Plot with Absolute Error Bound



1. Poles/Zeros after 400 days



2. Poles/Zeros after 1200 days
(after the water breakthrough)



Observability Analysis: Considerations

- Scale-independent observability analysis

$$\breve{A} = TAT^{-1} \qquad \breve{C} = CT^{-1}$$

$$\breve{B} = TB \qquad \breve{D} = D$$

- Correcting the dynamics for the different updating timestep

$$\breve{A}_u = \prod_{i=0}^{t_u} \breve{A}^i \qquad \breve{B}_u = \sum_{i=0}^{t_u} \breve{A}^i \breve{B}$$

- Observability matrix

$$\mathcal{O} = \begin{bmatrix} \breve{C} \\ \breve{C}\breve{A}_u \\ \breve{C}\breve{A}_u^2 \\ \vdots \\ \breve{C}\breve{A}_u^{n-1} \end{bmatrix}$$

$$\mathcal{O} = U\Sigma V^T$$

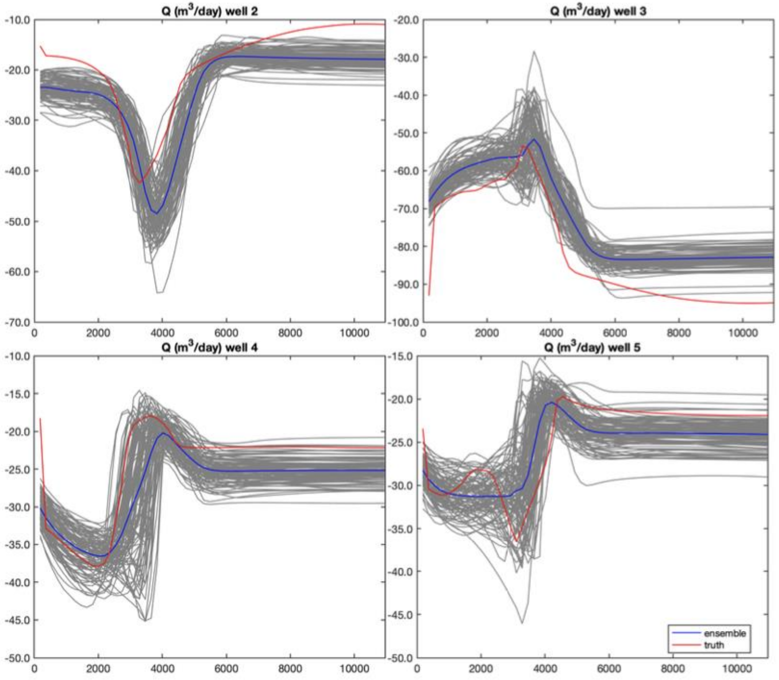
- Sampling in the high-observable directions

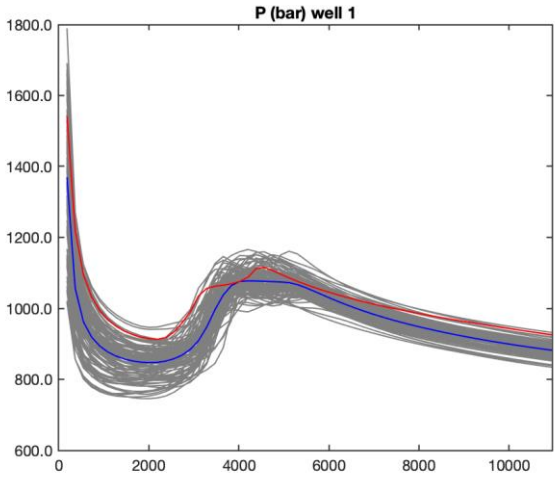
$$\hat{x}_j(t_0) = x_0 + \sqrt{\sigma_j} v_j$$

Observability-based Initiation

- This is implemented by normalizing the original initial ensemble matrix (the large ensemble) and premultiplying it by the observability matrix.
- Then, the ensemble members are selected such that they give the largest vector norm with respect to the original norm.

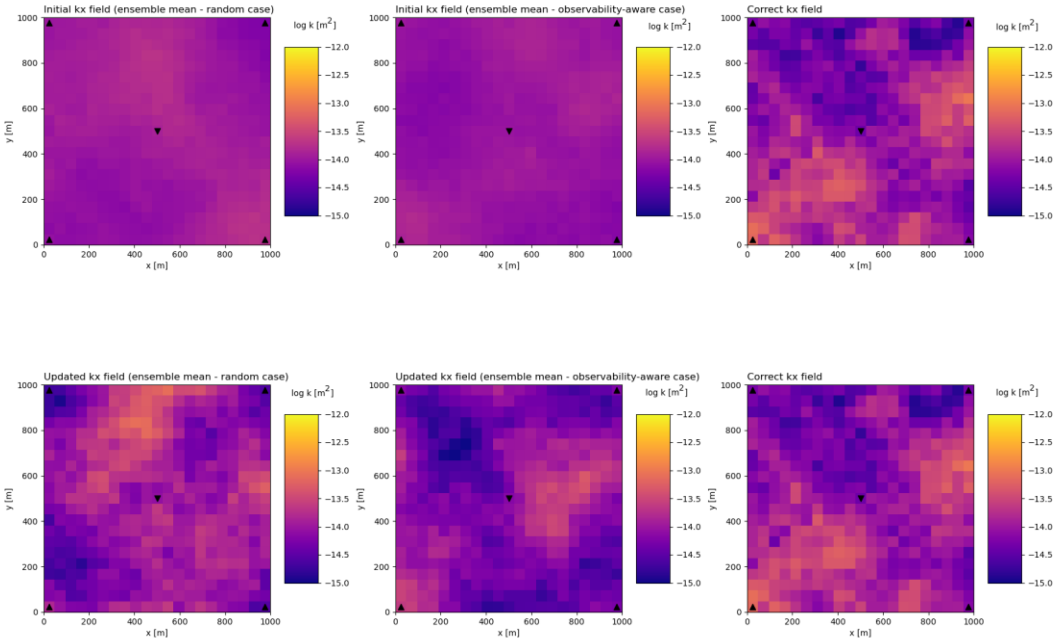
Observability-Aware EnKF: Results





Observability-Aware EnKF: Results

Seed	NRMSE
Shuffle 1	0.2391
Shuffle 2	0.1648
Shuffle 3	0.3242
Shuffle 4	0.1923
Shuffle 5	0.2510
Shuffle 6	0.1697
Shuffle 7	0.2311
Shuffle 8	0.1742
Shuffle 9	0.1629
Shuffle 10	0.3847
Mean	0.2294
Minimum	0.1648



$NRMSE(Obs. aware\ EnKF) = 0.1477$

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EnRML (iES)

$NRMSE(Obs.aware\ EnRML) = 0.2395$

Seed	NRMSE
Shuffle 1	0.2443
Shuffle 2	0.2434
Shuffle 3	0.2442
Shuffle 4	0.2597
Shuffle 5	0.2486
Shuffle 6	0.2558
Shuffle 7	0.2524
Shuffle 8	0.2450
Shuffle 9	0.2615
Shuffle 10	0.2500
Shuffle 11	0.2463
Shuffle 12	0.2507

Conclusions and Future Work

- Ensemble initiation is essential for the EnKF performance.
- Using observability analysis to select the initial realizations can improve the performance.
- However, we selected the most observable ones, but may be they are not the most important directions in optimization.
- Observability-based localization?