Multilevel Data Assimilation

moving towards realistic petroleum reservoir problems

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Outline

Problem Statement

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We consider the parameter estimation problem using spatially distributed data.

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Throughout the presentation, the parameters random vector is denoted by Z; the model forecasts vector, being a non-linear function of Z, is denoted by Y, $\mathrm{Y} = \mathcal{M}(\mathrm{Z})$; and the noisy data are denoted by D.

Motivation

Multilevel Data Assimilation

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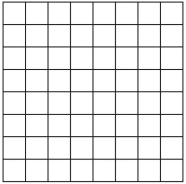
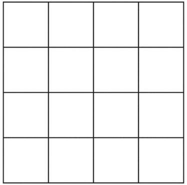
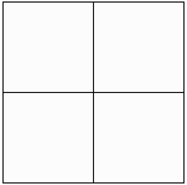
Multilevel data assimilation attempts to obtain a better balance Monte-Carlo and numerical errors by combining reservoir simulations with different fidelities

Multilevel Models


Finest Grid


Medium Coarse

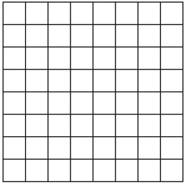

Coarsest Grid

Most Accurate Model

Med. Accurate Model

Least Accurate Model

Multilevel Models



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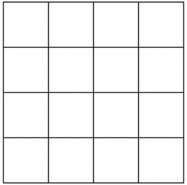

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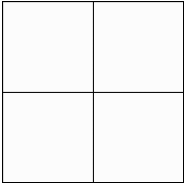

Coarsest Grid

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Med. Accurate Model

Least Accurate Model

Model Forecasts

Model Forecasts

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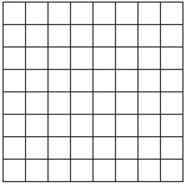
Multilevel Models



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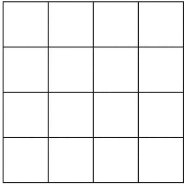

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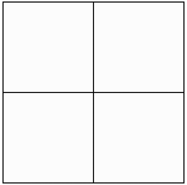


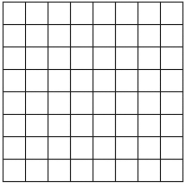
Coarsest Grid

Most Accurate Model | Med. Accurate Model | Least Accurate Model

Model Forecasts | Model Forecasts | Model Forecasts

Data

Multilevel Models and Multilevel Data

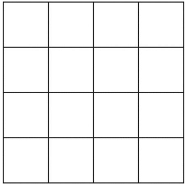


Finest Grid

Most Accurate Model

Model Forecasts

Data



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Model Forecasts

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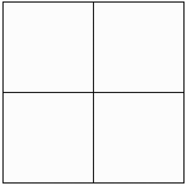

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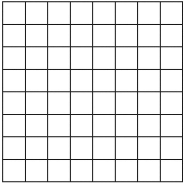
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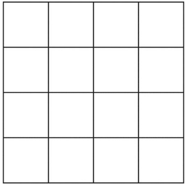
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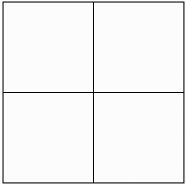
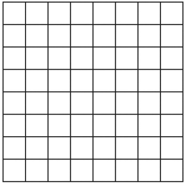

Coarsest Grid

$\mathcal{M}_L$	$\mathcal{M}_l$	$\mathcal{M}_1$
Y_L	Y_l	Y_1
D_L	D_l	D_1

Multilevel Models and Multilevel Data



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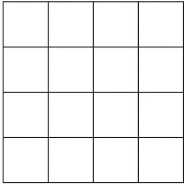

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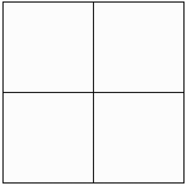


Coarsest Grid

$\mathcal{M}_L$ $\quad$ $\mathcal{M}_l$ $\quad$ $\mathcal{M}_1$

Y_L $\quad$ Y_l $\quad$ Y_1

D_L $\quad$ D_l $\quad$ D_1

$$D_l = U_L^l D_L$$

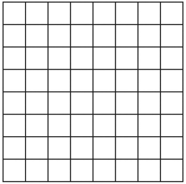
Developed Multilevel Methods

Simultaneous MLDA Algorithms

ML Modeling Error Correction Schemes

Sequential MLDA Algorithms

Simultaneous Multilevel Data Assimilation



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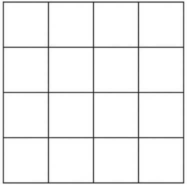

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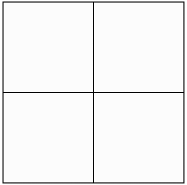

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Simultaneous MLDA

Nezhadali, M., et al. "A Novel Approach to Multilevel Data Assimilation." ECMOR XVII. Vol. 2020. No. 1. European Association of Geoscientists & Engineers, 2020.

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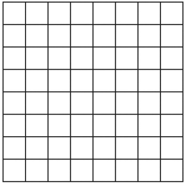

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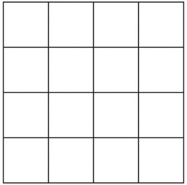

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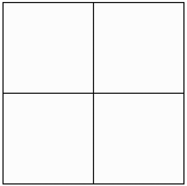


Coarsest Grid

$$D_l = U_L^l D_L$$

$$\zeta_l = U_L^l Y_L - Y_l$$

$\zeta = 0$

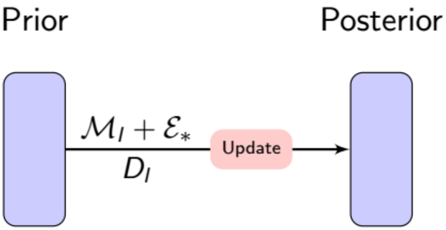
ζ

ζ

$$\varepsilon \approx \zeta$$

NORCE

ML Modeling Error Correction Schemes



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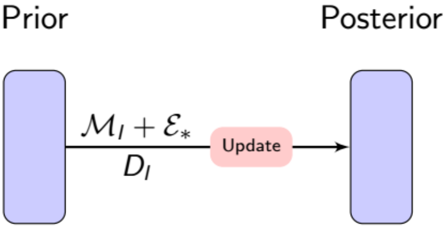

Nezhadali, Mohammad, et al. "Multilevel Assimilation of Inverted Seismic Data With Correction for Multilevel Modeling Error." (2021).

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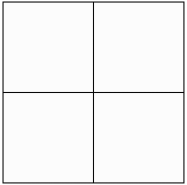
Sequential Multilevel Data Assimilation



Coarsest Grid

$\mathcal{M}_1$

Y_1

D_1

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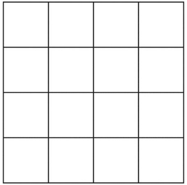
Sequential Multilevel Data Assimilation



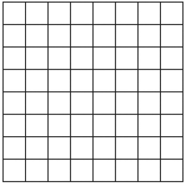
Medium Coarse

$\mathcal{M}_l$

Y_l

D_l

Sequential Multilevel Data Assimilation



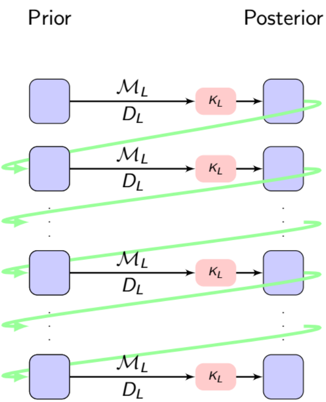
Finest Grid

$\mathcal{M}_L$

Y_L

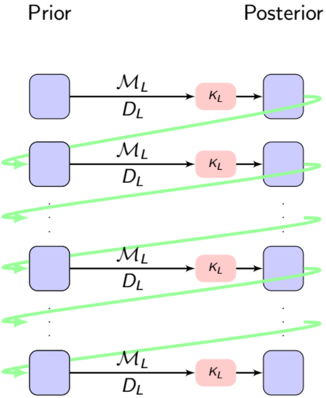
D_L

Sequential MLDA vs. ESMDA

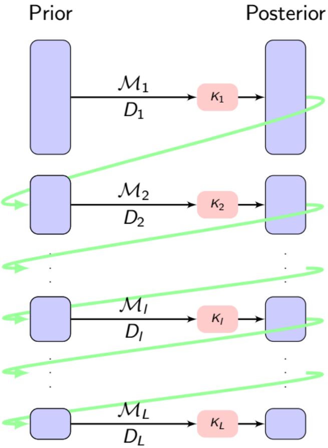


ESMDA

Sequential MLDA vs. ESMDA



ESMDA



SMLES

Numerical Experiments

Two experiments pertaining to subsurface flow are presented.

Unknown parameter field: flow conductivity

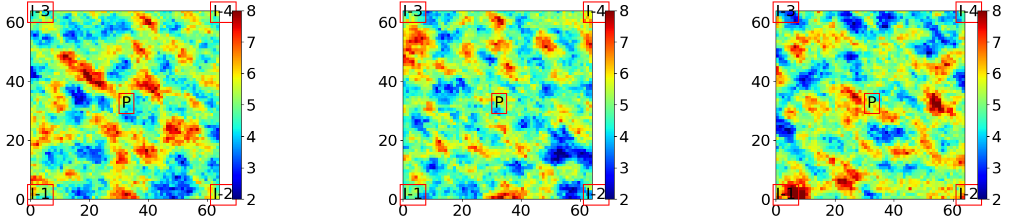
Observation data: grid data at three different times

Each experiment has four algorithm runs:
SMLES, IES-LOC, ESMDA-LOC, ESMDA-REF

The gold standard for comparison will be vanilla ESMDA with an exceedingly large ensemble size (10000 members). This is run to obtain the best DA results that can be obtained by ESMDA (ESMDA-REF).

Case I

Prior Model–flow conductivity



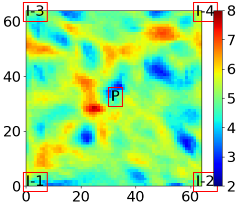
Prior Samples–log K

variance	mean	range	aniso	angle	type
1	5	10	0.7	-30	spherical

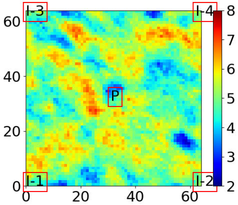
Variogram for prior draw

Case I–Posterior Parameters

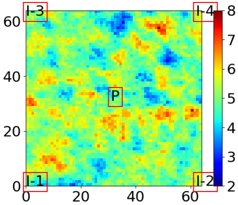
Mean field



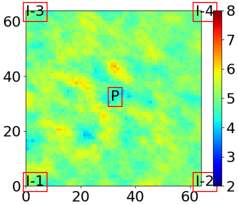
(a) ESMDA-REF



(b) SMLES



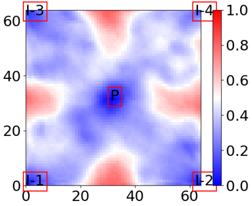
(c) ESMDA-LOC



(d) IES-LOC

Case I–Posterior Parameters

Variance field



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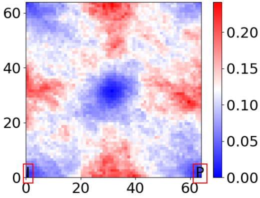
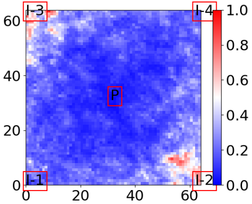
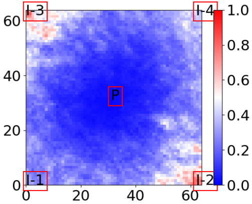


(b) SMLES*



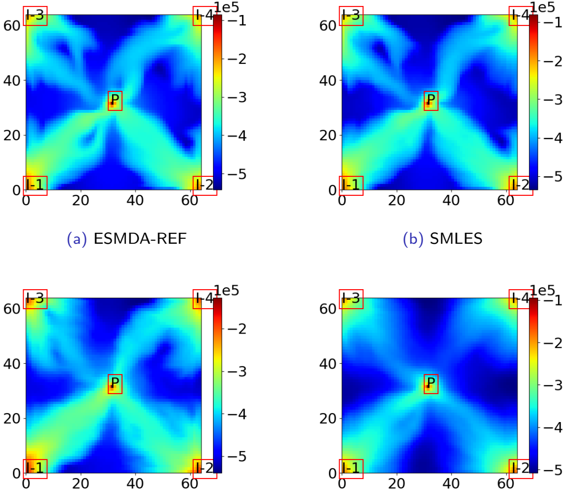
(c) ESMDA-LOC



(d) IES-LOC

Case I–Posterior Model Forecasts

Mean field

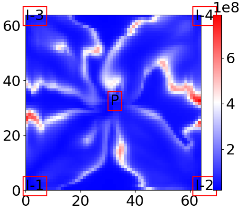


(a) ESMDA-REF (b) SMLES

(c) ESMDA-LOC (d) IES-LOC

Case I–Posterior Model Forecasts

Variance field



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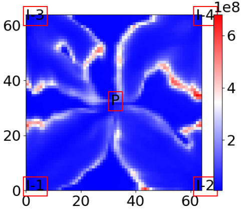

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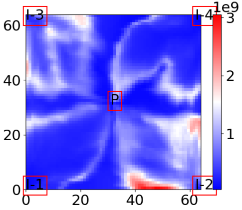

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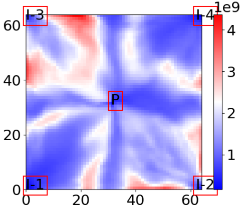
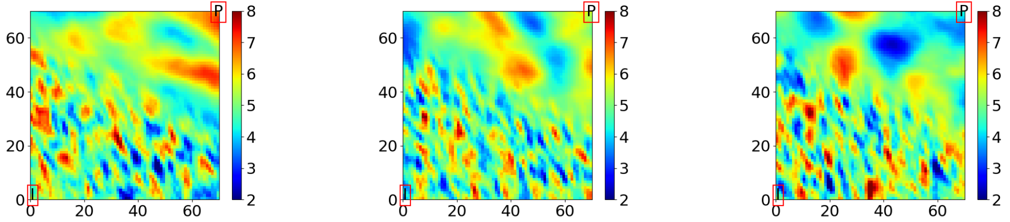


(d) IES-LOC

Case II

Prior Model–Flow conductivity



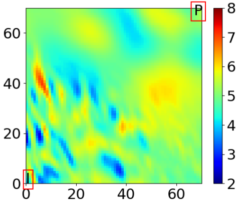
Prior Samples–log K

The variograms of permeability zones for prior draw

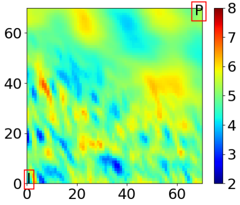
	variance	mean	range	ratio	angle	type
Variogram 1	1	5	30	0.7	-30	cubic
Variogram 2	1	5	10	0.4	-70	cubic

Case II–Posterior Parameters

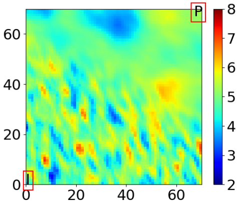
mean field



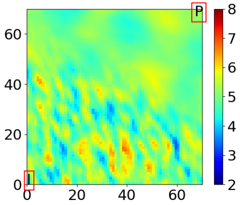
(a) ESMDA-REF



(b) SMLES



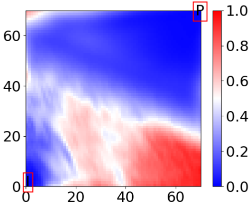
(c) ESMDA-LOC



(d) IES-LOC

Case II–Posterior Parameters

variance field



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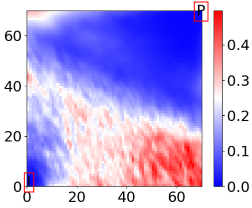

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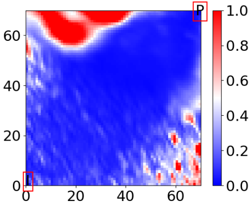

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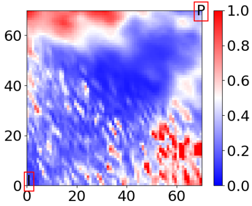
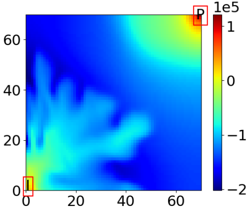


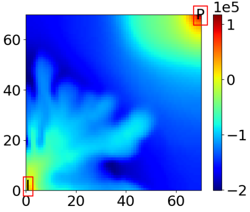
(d) IES-LOC

Case II–Posterior Model Forecasts

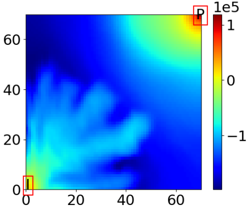
mean field



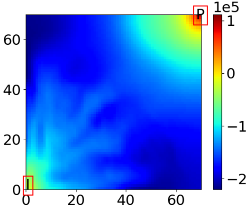
(a) ESMDA-REF



(b) SMLES



(c) ESMDA-LOC



(d) IES-LOC

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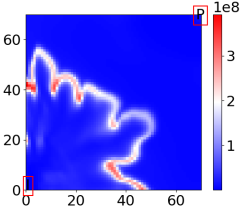
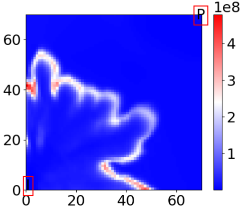
Case II–Posterior Model Forecasts

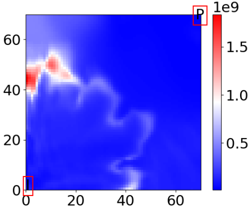
variance field



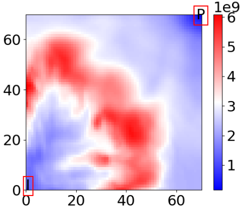
(a) ESMDA-REF



(b) SMLES



(c) ESMDA-LOC



(d) IES-LOC

Current and Further Work

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- Inversion of large correlated covariance matrices with the help of SPDEs
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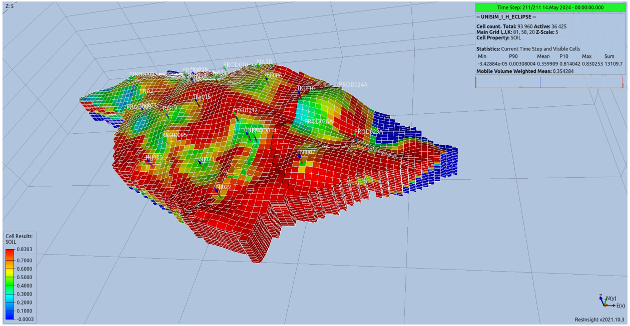

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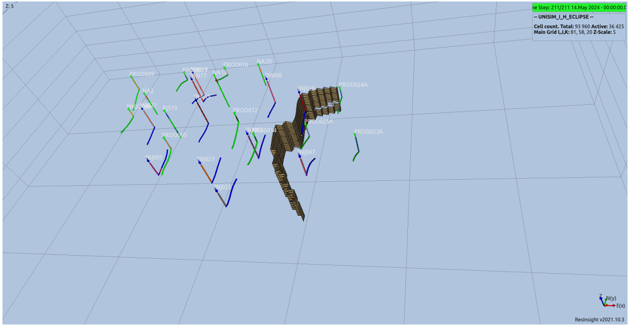

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Thanks for your attention