Uncertainty quantification for source reconstruction of ^{137}Cs released during the Fukushima accident

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Fukushima-Daiichi accident

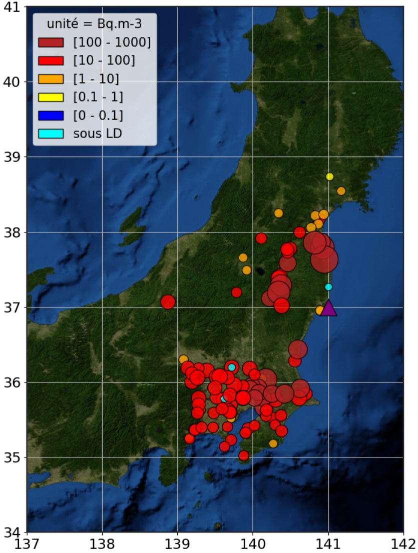
Fukushima-Daiichi nuclear disaster in March 2011.

Release of large quantities of radionuclides, including ^{137}Cs:

- on three weeks,
- with an important temporal variability.

Fukushima-Daiichi ^{137}Cs observations

- $>$ 14,000 hourly air concentration measurements between $11/03/2011$ and $23/03/2011$
- $>$ 1,000 deposition measurements;
- Use of the Eulerian transport model ldX represented by a linear observation operator $\mathbf{H}$
- Meteorological data: ECMWF OD (0.125°, 3h);



Observation equation

Source vector (describing the source of the releases)

Observation operator

Observation error

$$\boxed{\mathbf{y} = \underbrace{\mathbf{H}\mathbf{x}} + \epsilon}$$

Observations vector

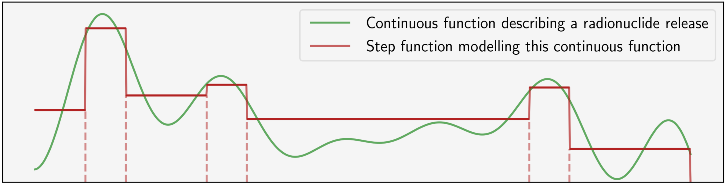
Radionuclide transport modelling
Modelled predictions for a given source

- All variables describing the source of the release (height, coordinates,...) are assumed to be known, except for
  - the source term $\mathbf{q}$ or vector of constant release rates of size $\mathrm{N}_{\mathrm{imp}}$ (number of pulses).

But how to characterise q ?

Representation of the continuous release by a step function

The source term $\mathbf{q}$ needs to be well characterised



Representation of the release $\rightarrow$ solving a trade-off between

- bias (too simple model, insufficient to learn correctly from data),
- variance (overfitting or overinterpretation of the data).

Inverse the source $\longrightarrow$ inverse the parametrisation of the source

Bayesian inverse modelling

Bayes' formula

Bayes' formula, with $\mathbf{x}$ the vector of variables characterising the source and $\mathbf{y}$ the observations is written

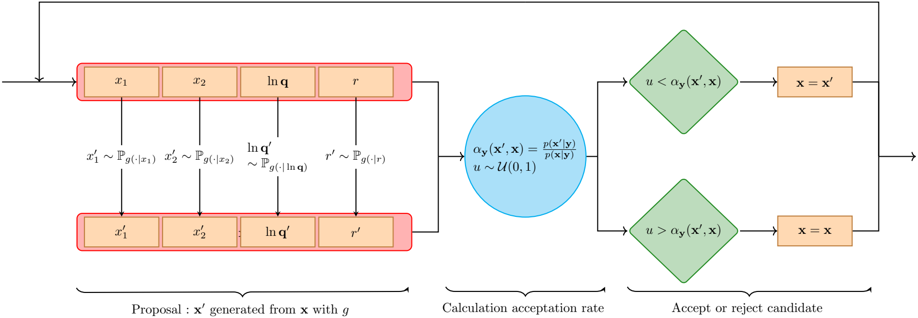
$$\overbrace{p(\mathbf{x}|\mathbf{y})}^{\text{Posterior}} = \frac{\overbrace{p(\mathbf{y}|\mathbf{x})}^{\text{Likelihood}}\ \overbrace{p(\mathbf{x})}^{\text{Prior}}}{\underbrace{p(\mathbf{y})}_{\text{Evidence}}} \propto p(\mathbf{y}|\mathbf{x})p(\mathbf{x}).$$

$\mathbf{y}|\mathbf{x}$ diagnostics the difference between the observations $\mathbf{y}$ and the dispersion model results computed out of the source $\mathbf{x}$.

Source vector = variables of interest to sample:

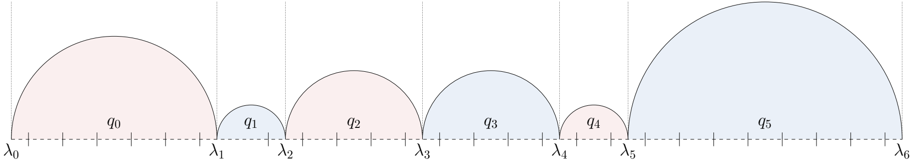
- release rates $\mathbf{q}$;
- observation error scale matrix $\mathbf{R}$;
- source prior scale matrix $\mathbf{B}$.

A popular MCMC algorithm: Metropolis-Hastings



Model selection: Reversible-Jump MCMC

- The constant release rates q_i are separated by "edges" λ_i.
- The evolution of the release is modelled by a specific *partition* of edges.



The transdimensional partition of edges $\Lambda = \{\lambda_0, \lambda_1, ...\}$ is integrated as a variables ensemble to the MCMC procedure:

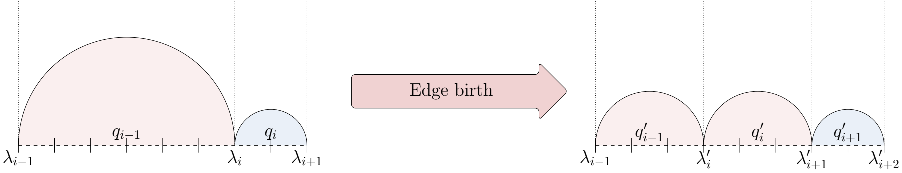
$$\mathbf{x} = (\ln \mathbf{q}, \mathbf{R}) \rightarrow \mathbf{x} = (\ln \mathbf{q}, \mathbf{R}, \Lambda)$$

$\Longrightarrow$ Use of the Reversible Jump MCMC[1].

[1] Green 1995; Liu et al. 2017.

RJ-MCMC: random transdimensional jumps

An example of transdimensional procedure : the "edge" birth[2].



$$\mathbf{x} = (\lambda_0, ..., \lambda_n, q_0, ..., q_k, ...) \longleftrightarrow (\lambda_0, ..., \lambda'_i, ..., \lambda_n, q_0, ..., q_k - u_{\ln q}, q_k + u_{\ln q}, ...) = \mathbf{x}'$$

- Dimension change of $\mathbf{x}$ $\rightarrow$ addition of terms ensuring the *detailed balance* in the MCMC algorithm;
- Need to define new priors and transition probabilities on the edge variables and release rates.

[2] Bodin and Sambridge 2009.

Redefining the observation error scale matrix

- Need to integrate more information.
  - Factor both concentration and deposition measurements into a Bayesian sampling
  - Take in account spatial distances between air concentration measurements

$$\mathbf{R} = \begin{bmatrix} r_{c,i} & 0 & ... & ... & ... & 0 \\ 0 & r_{c,j} & 0 & ... & ... & ... \\ ... & 0 & r_{c,i} & ... & ... & ... \\ ... & ... & 0 & ... & 0 & ... \\ ... & ... & ... & ... & r_d & 0 \\ 0 & ... & ... & ... & 0 & r_d \end{bmatrix}. \quad (1)$$

Vector to sample:

$$\mathbf{x} = \left(\ln \mathbf{q}, r_{c,1}, \ldots, r_{c,i}, ..., r_d, (\lambda_1, \ldots, \lambda_{\mathrm{N_{imp}}-1})\right) \quad (2)$$

Definition of the distributions

- Likelihood $\mathbf{y}|\mathbf{x} \sim \log -$Cauchy with scale $\mathbf{R}$[3].

$$p(\mathbf{y}|\mathbf{x}) \propto \prod_{i=1}^{\text{Nobs}} \frac{1}{(y_i + y_t)\pi\sqrt{r_i}\left(1 + \frac{(\ln(y_i+y_t)-\ln(\mathbf{Hx}_i+y_t))^2}{r_i}\right)} \tag{3}$$

  - efficient to manage observations of different orders of magnitude;
  - to manage the observations close to zero → we add a threshold term[4] y_t;

- Prior definitions
  - Uniform priors on the scale parameters;
  - Exponential prior on edges to penalise too complex models:

$$p(\lambda_1, \dots, \lambda_k) = \begin{cases} \frac{e^{-k}}{\sum_{i=1}^{\mathrm{N_{b,max}}} \frac{\mathrm{N_{b,max}}!}{i!(\mathrm{N_{b,max}}-i)!} e^{-i}}, & \text{if } k \in \{1, 2, \dots, \mathrm{N_{b,max}}\}; \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

[3] Dumont Le Brazidec et al. 2021.

[4] Liu et al. 2017.

Prior on the release rates

- Some release pulses are not constrained by the observations.
- Folded gaussian prior is set with $\mathbf{B}$ parametrised with parameters b_c for constrained pulses and b_nc for non-constrained pulses
- $\mathbf{B}$ is adapted for our case: pulses sampled are combinations of hourly pulses

$$p(\ln \mathbf{q}|\mathrm{N_{imp}}) = \prod_{i=1}^{\mathrm{N_{imp}}} \sqrt{\frac{2}{\pi(w_{\mathrm{c},i}b_\mathrm{c} + w_{\mathrm{nc},i}b_\mathrm{nc})}} \left(e^{-\frac{(\ln q_i)^2}{2(w_{\mathrm{c},i}b_\mathrm{c} + w_{\mathrm{nc},i}b_\mathrm{nc})}} \right)$$

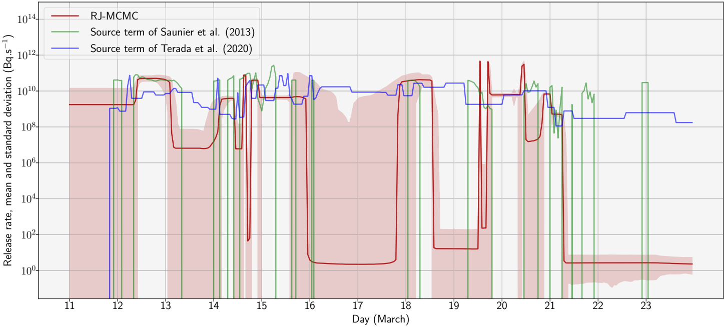
- $\mathrm{N_{imp}}$ is the number of pulses at a certain RJ-MCMC iteration
  - characterises the grid on which $\ln \mathbf{q}$ is defined.
  - $\mathrm{N_{imp}} = \mathrm{N_b} - 1$ ($\mathrm{N_b}$ is the number of edges)
- this prior also constrains the model's complexity

We sample:

$$\mathbf{x} = \left(\ln \mathbf{q}, r_{c,1}, \ldots, r_{c,i}, ..., r_d, b_c, (\lambda_1, \ldots, \lambda_{\mathrm{N_{imp}}-1})\right) \tag{5}$$

Fukushima-Daiichi ^{137}Cs release rate reconstruction

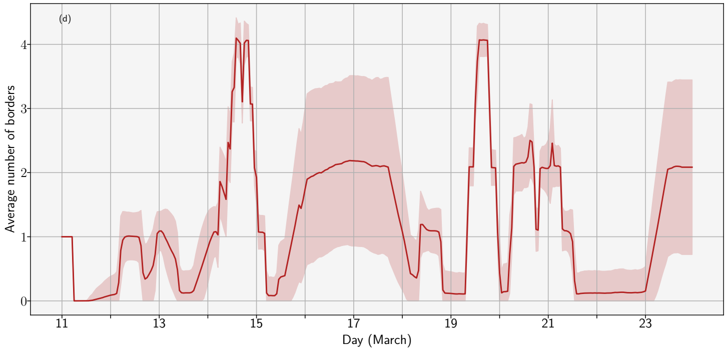
Fukushima-Daiichi ^{137}Cs release rate evolution (and corresponding variance) in Bq.s^{-1}.



- Several release episodes (12-13 march, 14-16 march, 18 march, 19-21 march) of diverse variabilities;
- Large *variability of the variability* → proves RJ-MCMC pertinence.

Fukushima-Daiichi ^{137}Cs variability reconstruction

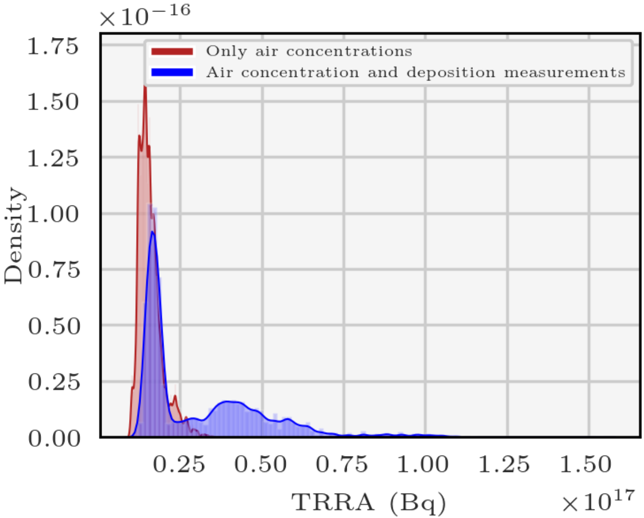
Evolution of the number of edges sampled around each hour (and corresponding variance).



- High variability between march 14 and 15, and march 19 and 21: periods of intense release;
- Low variability elsewhere (apart from artefacts in non-constraints periods).

Fukushima-Daiichi ^{137}Cs source term reconstruction

Reconstruction of the total ^{137}Cs release with or without deposition measurements



Conclusions

- Complex releases → high variability and high variability of the variability
- Model of such a release complicated to define because:
  - might be at some periods too simple
    → *bias* errors
  - might be at some periods too complicated
    → overfitting + *variance* errors;
- Use of RJ-MCMC allows to
  - reconstruct the best model by solving the bias-variance trade-off,
  - and thus, better estimate the uncertainties related to the release representation.