Multivariate Ensemble Sensitivity Analysis for Understanding Dynamics of Extreme Weather Events

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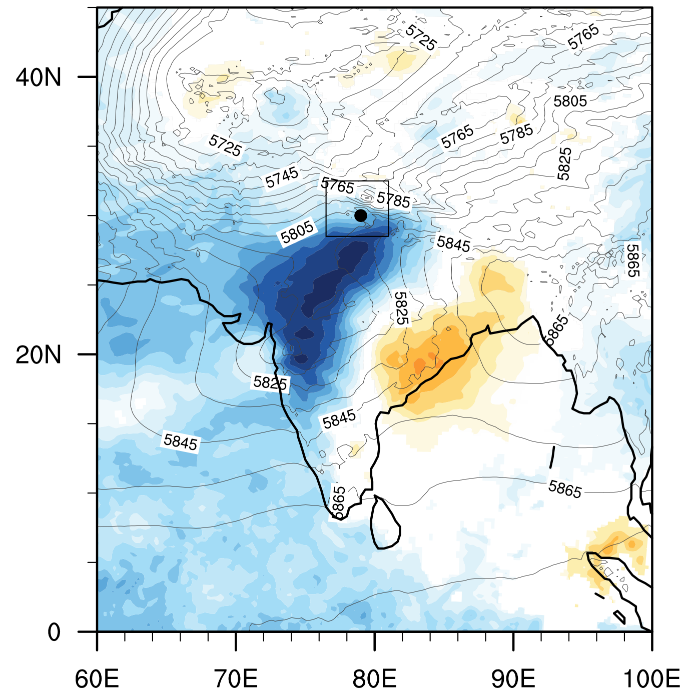
Sensitivity Analysis

How does change in a set of initial state variable affect the change a forecast metric ?

- Understand the predictability of weather events
- To propose optimal locations of observation network that may benefit data assimilation systems

Sensitivity Analysis

- Adjoint Sensitivity uses single integration of adjoint models
- Ensemble Sensitivity employs ensemble statistics to estimate the sensitivity of initial conditions to forecast metric



Sensitivity of an extreme precipitation forecast to geopotential height in initial condition over India (from George and Kutty, 2020)

Ensemble Sensitivity Analysis (ESA)

The expression of ensemble sensitivity can be derived from an ordinary least square problem

$$\mathbf{J}_e = [\mathbf{X}^a]^{\mathrm{T}} \boldsymbol{\beta} + \boldsymbol{\varepsilon},$$

The solution of $\hat{\boldsymbol{\beta}}$ gives the linear statistical estimate of sensitivity

$$\hat{\boldsymbol{\beta}} = \frac{\partial J_e}{\partial \mathbf{x}^a}$$

Here $\mathbf{X}^a$ and $\mathbf{J}_e$ are the matrix and vector obtained by collecting the ensembles of size P x K and 1 x K, respectively, with the ensemble mean removed (Torn and Hakim, 2008)

Ensemble Sensitivity Analysis (ESA)

Ancell and Hakim (2007) ignored the contributions from off diagonal components in $\mathbf{P}^a$ in the calculation of ensemble sensitivity where $\mathbf{P}^a$ is $\mathbf{X}^a[\mathbf{X}^a]^{\mathrm{T}}$, ensemble estimated analysis error covariance

$$\frac{\partial J}{\partial \mathbf{x}^a} = \mathbf{X}^a \mathbf{J}_e (\mathbf{P}^a)^{-1} \approx \mathbf{X}^a \mathbf{J}_e (\mathbf{D}^a)^{-1}$$

Assume that assimilation a hypothetical observation is leads to analysis increment equal to ensemble spread σ_p , the corresponding change in forecast response function is as follows

$$\delta J_p^u = \sigma_p \times \frac{\partial J}{\partial x_p}, \quad p = 1, 2, \ldots, P,$$

Case Study

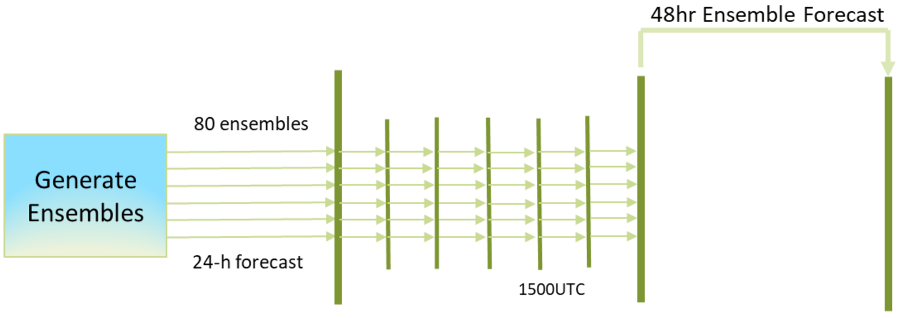
Extreme Rainfall over Uttarakhand in June 2013

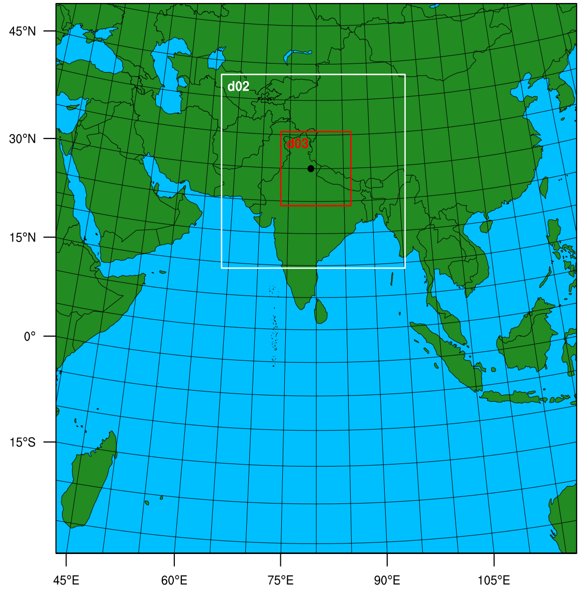
- *Strong synoptic forcing caused massive destruction to life and properties.*
- *Over 4000 villages were affected, and the death toll exceeded 5000*

Using Ensemble Sensitivity Analysis key synoptic and mesoscale factors that led to the event has been investigated

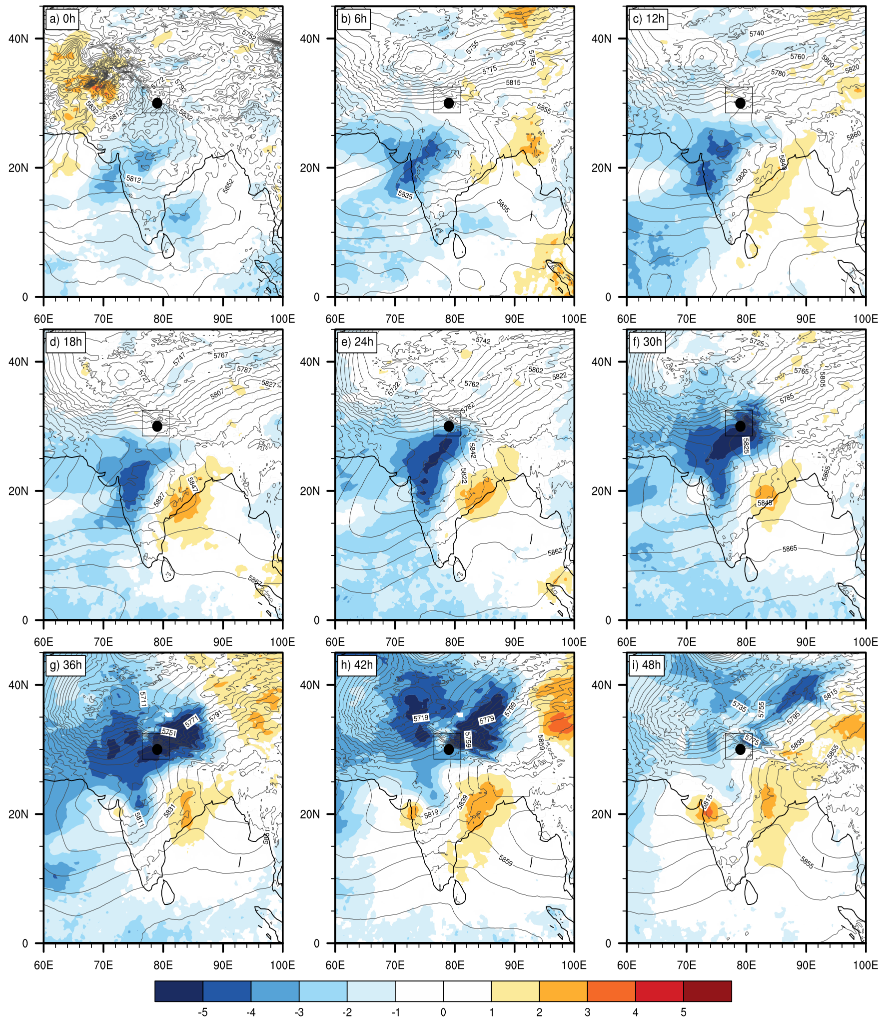
Experimental Design

- 80 member Ensemble Data Assimilation with Data Assimilation Research Testbed (DART)
- Weather Research and Forecast Model
- Two-way nesting for two nested domain



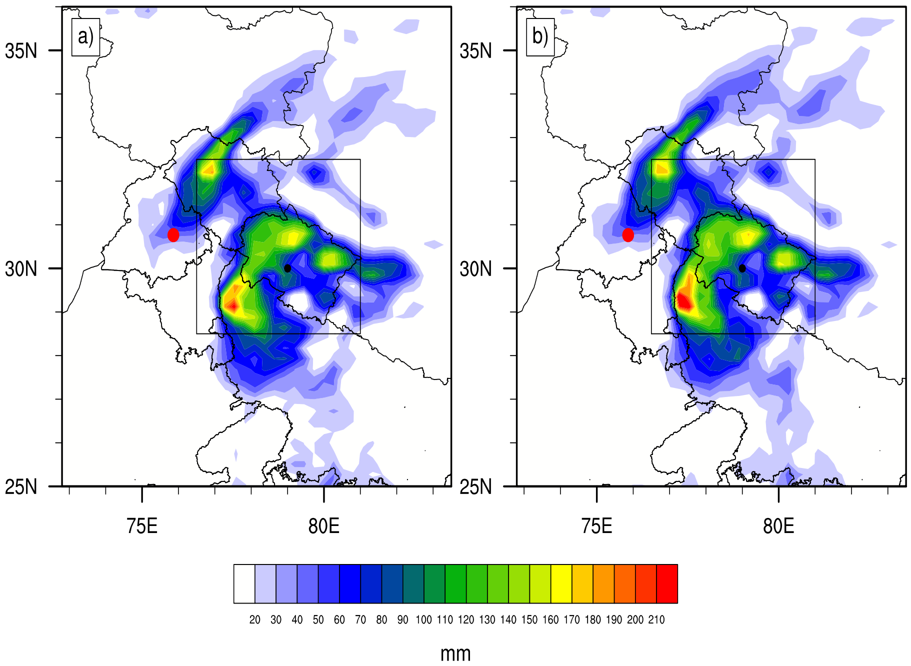


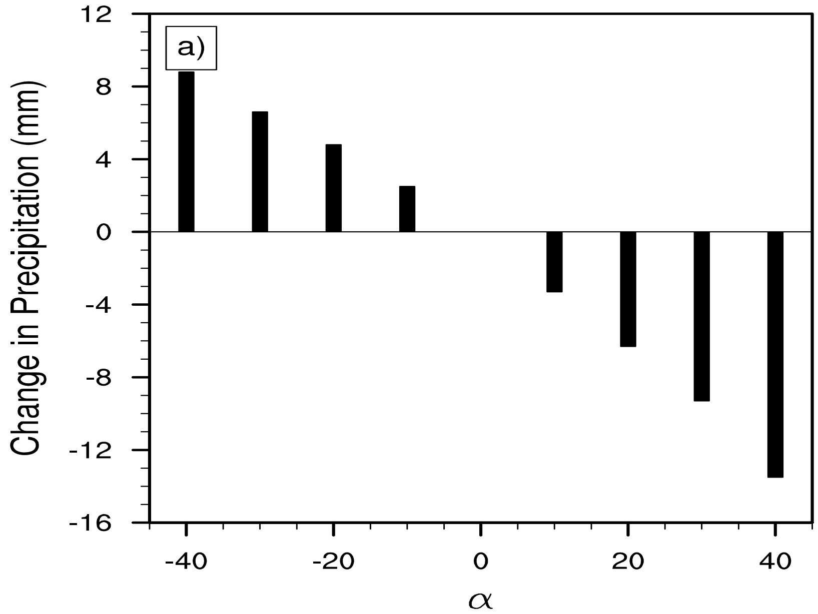
Results



- Sensitivity of precipitation to geopotential height at 500 hPa level
- A sensitivity dipole to the south of the rainfall location is observed
- Southward extension of mid-latitude trough intensified the flow of moist air towards the response function region

Effect of synthetic observation





$$\boldsymbol{x}_i^p = \boldsymbol{x}_i^a + \frac{\partial x_i^a}{\partial x_s}\alpha$$

Increase in the geopotential height at the sensitive grid point decreases the precipitation in the box

Multivariate ESA

By ignoring off-diagonal components, the sensitivity of individual state elements will be overestimated.

This will be more evident when addressing a weakly forced scenario or an unbalanced dynamics of a mesoscale flow

Hacker and Lei (2015) proposed a multivariate sensitivity that compute the sensitivity with a multivariate regression that retains full covariance matrix

Multivariate ESA

Solving multivariate regression coefficient yields

$$\boldsymbol{\beta} = \frac{\partial J}{\partial \mathbf{x}} = \mathbf{X}(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{J}_e$$

In which $\mathbf{X}$ is a P x K matrix composed of ensemble perturbations denotes multivariate ensemble sensitivity $\boldsymbol{\beta}$

$$\mathbf{x}_1^e, \mathbf{x}_2^e, \ldots, \mathbf{x}_P^e$$

Multivariate ESA

The estimated change in forecast response using multivariate ensemble sensitivity with localized analysis increment is as follows

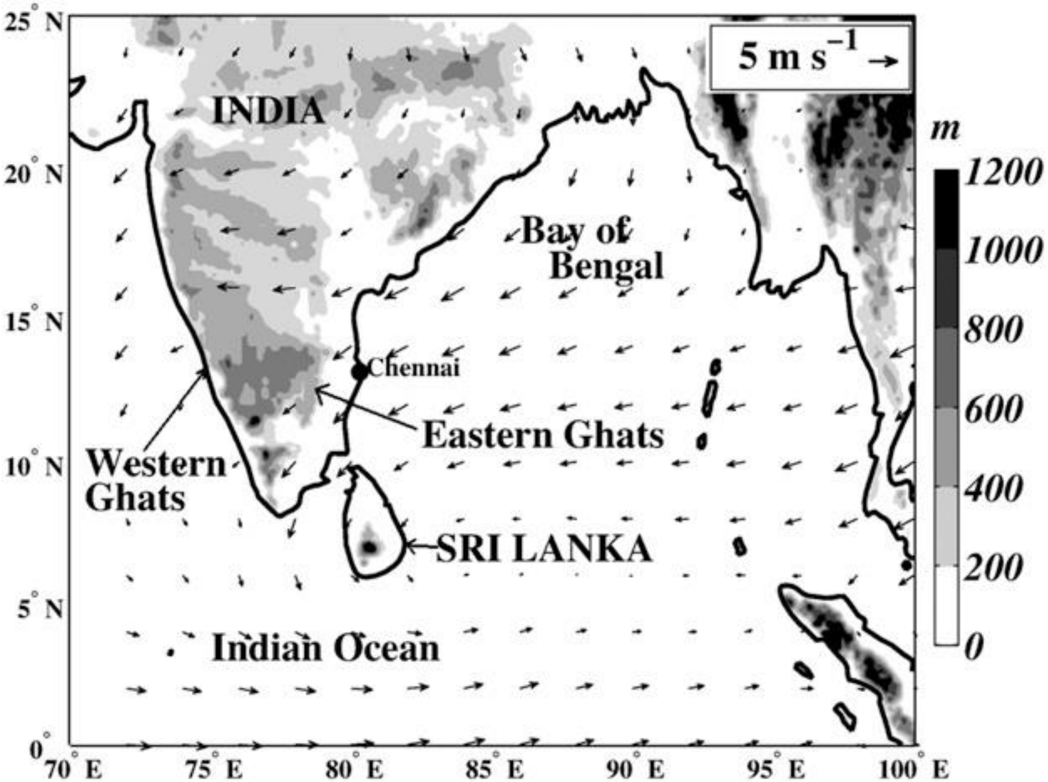
$$\delta J_p^m = \left(\frac{\partial J}{\partial \mathbf{x}}\right)^{\mathrm{T}} \boldsymbol{\rho} \circ (\delta \mathbf{x})$$

The analysis increment $\delta\mathbf{x}$ can be expressed as

$$\delta(x_i) = \begin{cases} \sigma_p, & i = p, \\ \sigma_p \times \dfrac{\mathrm{cov}(\mathbf{x}_i^e, \mathbf{x}_p^e)}{\mathrm{var}(\mathbf{x}_i^e)}, & i = 1, \ldots, p-1, p+1, \ldots, P. \end{cases}$$

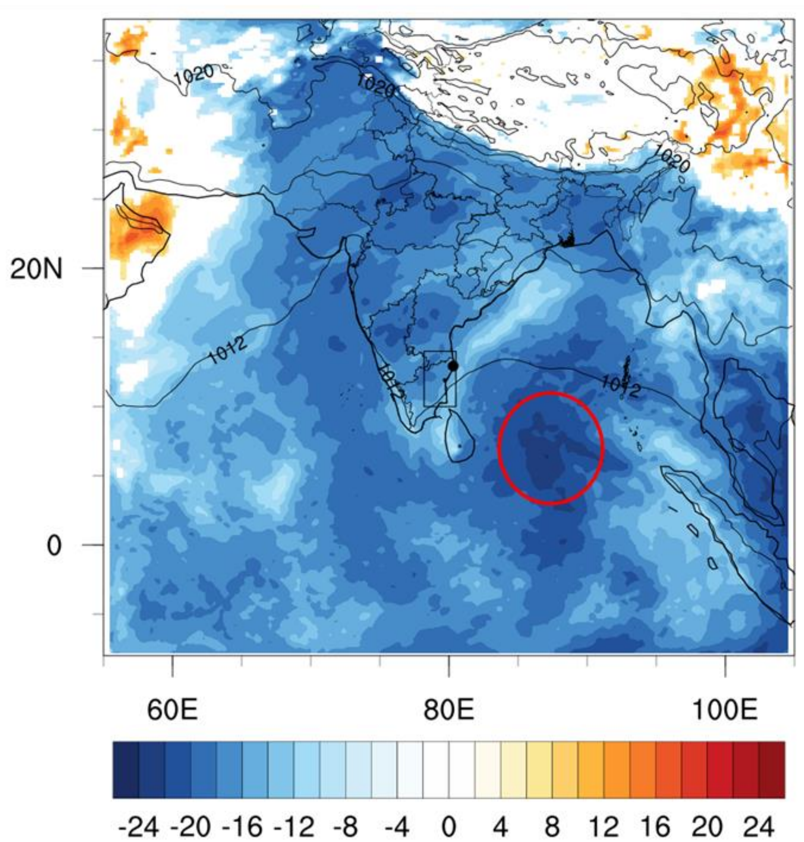
Extreme Rainfall over Chennai in 2015

- *50 cms of rainfall in 24 hours*
- *Death toll was over 250 people*

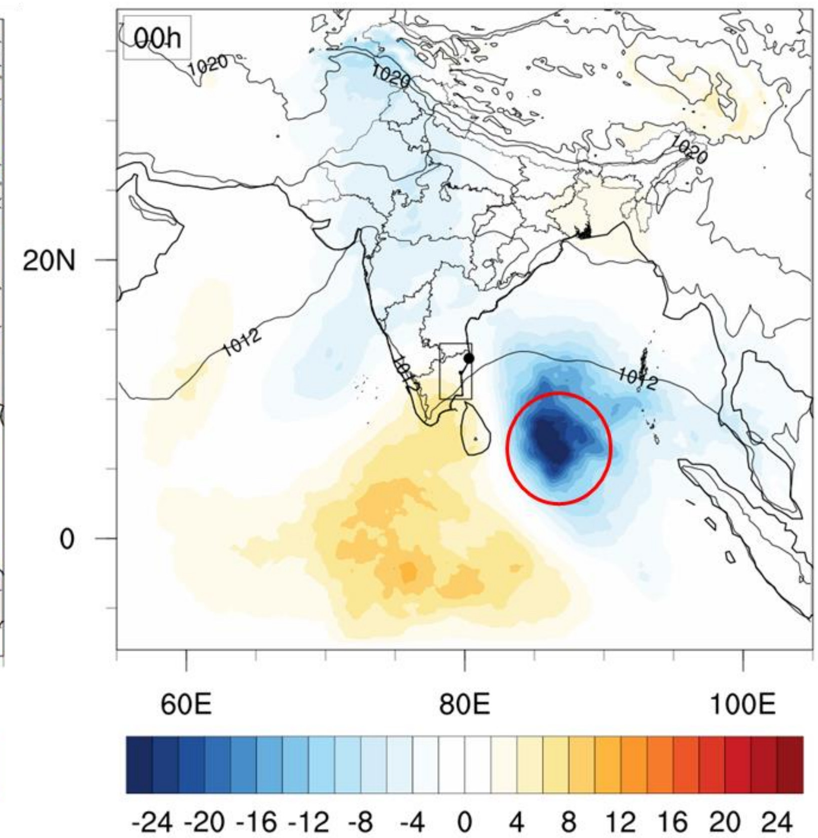


Univariate vs. Multivariate

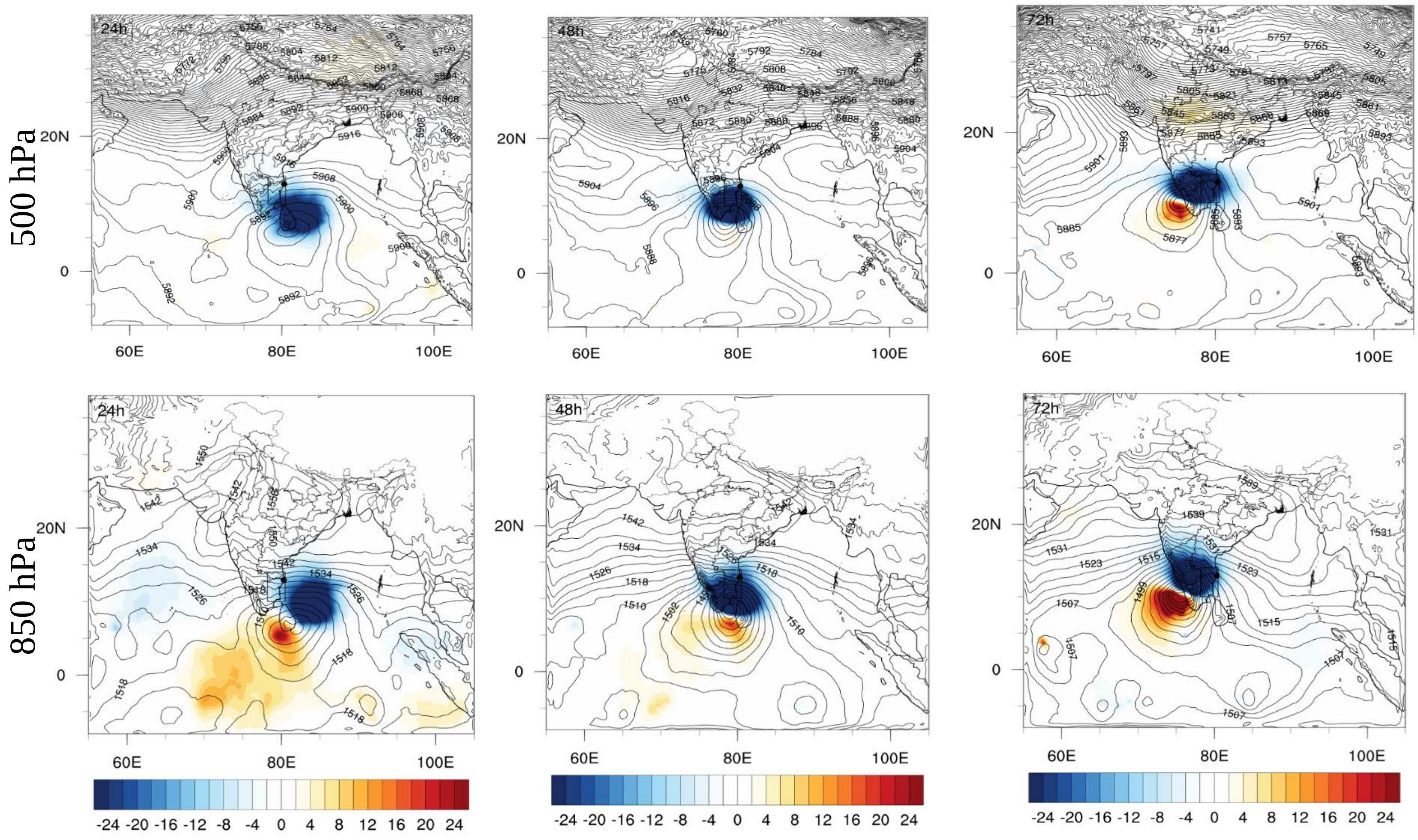
UV



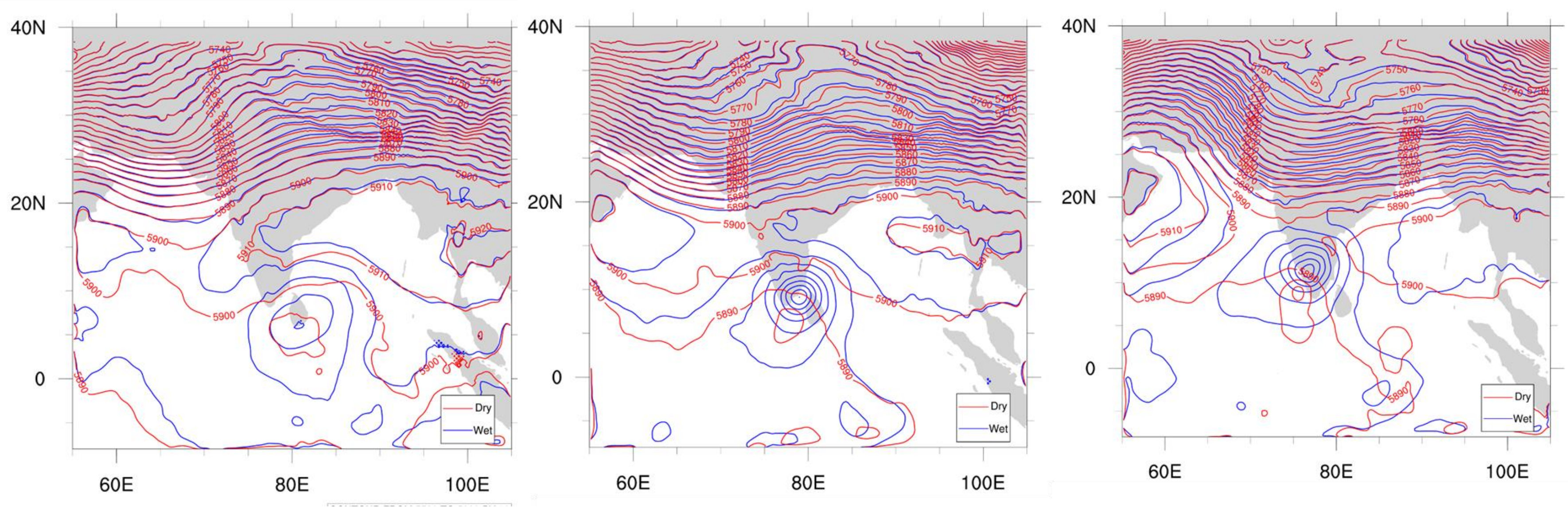
MV



Dynamics using Multivariate



Composite difference of Wet and Dry ensembles



Conclusions

- The univariate ensemble sensitivity uses diagonal approximation while multivariate ensemble sensitivity retains full covariance matrix when computing multivariate regression

- Initial results indicate that the univariate approach overestimates the sensitivity values as compared to multivariate approach

Thank You

References

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Ren, S., Lei, L., Tan, Z., & Zhang, Y. (2019). Multivariate Ensemble Sensitivity Analysis for Super Typhoon Haiyan (2013), *Monthly Weather Review*, *147*(9), 3467-3480.

Torn, Ryan D., and Gregory J. Hakim. "Ensemble-based sensitivity analysis." *Monthly Weather Review* 136.2 (2008): 663-677.

Regress Perturbation

Perturbations are applied to the analysis ensembles in the most sensitive regions and integrated the model forward from the perturbed initial conditions

$$\boldsymbol{x}_i^p = \boldsymbol{x}_i^a + \frac{\partial x_i^a}{\partial x_s}\alpha$$

$$\frac{\partial x_i^a}{\partial \boldsymbol{x}_s} = \frac{cov\left(\boldsymbol{x}_i^a, \boldsymbol{x}_s\right)}{var\left(\boldsymbol{x}_s\right)}$$

Perturbations of geopotential at the most sensitive region is regressed to the remaining state variables