Ensemble based data assimilation for a multi-compartment porous media model

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- Data assimilation in medicine
- Contrast enhanced dynamic MRI
- Multi-compartment porous media model for blood flow
- Data assimilation
- Case studies
- Conclusion

Blood flow simulation

- A. L. Marsden: “Cardiovascular blood flow simulation. From computation to clinic.” SIAM News, 48 (10), 2015.
  - “There is a rising interest in clinical data assimilation and uncertainty quantification in cardiovascular simulations.”

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- Boundary value problem
- Data source: Medical imaging
- Using variational methods
- Forward modeling is computationally intensive
- Adjoints available

Ensemble based approach

S. Pagani, A. Manzoni and A. Quarteroni: ”A reduced basis ensemble Kalman filter for state/parameter identification in large-scale nonlinear dynamical systems”. Tech. report 18.2016 Mathematics Institute of Computational Science and Engineering, École polytechnique fédérale de Lausanne

Data assimilation in oncology - I

> TE Yankeelov et. al.: “Toward a science of tumor forecasting for clinical oncology,” Cancer Research, 75(6), 2015.

- “We propose that the quantitative cancer biology community makes a concerted effort to apply lessons from weather forecasting to develop an analogous methodology for predicting and evaluating tumor growth and treatment response.”

Data assimilation in oncology - II

- EJ Kostelic et. al.: “Accurate state estimation from uncertain data and models: an application of data assimilation to mathematical models of human brain tumors.” Biology Direct, 6:64, 2011.
  - Synthetic study using two models for glioblastoma, MRI data as observations.
  - Local ensemble transform Kalman filter

Contrast enhanced dynamic MRI

Contrast enhanced dynamic MRI

- Gadolinium Contrast Medium (MRI Contrast agents)

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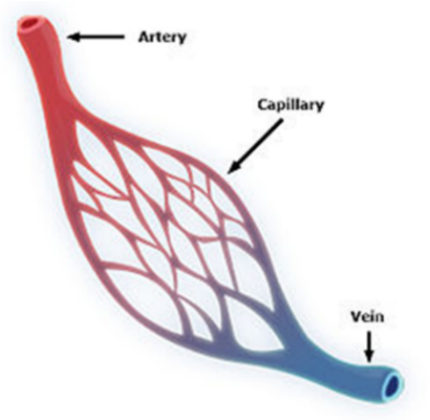
https://commons.wikimedia.org/w/index.php?curid=3546051

Contrast enhanced dynamic MRI

- Gadolinium Contrast Medium (MRI Contrast agents)
- Determine blood perfusion



https://en.wikipedia.org/wiki/Capillary

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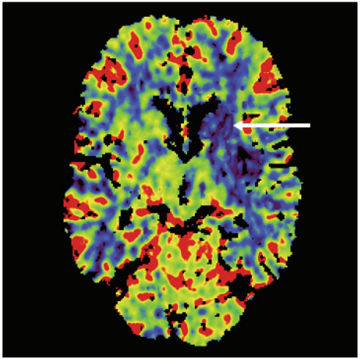
- Diagnosis and treatment of cancer, strokes



Fieselmann et. al., 2011,
Int. Journal of Biomedical Computing
doi:10.1155/2011/467563

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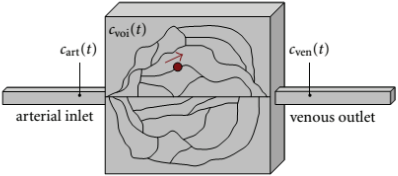

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Contrast enhanced dynamic MRI

- Gadolinium Contrast Medium (MRI Contrast agents)
- Determine blood perfusion
- Diagnosis and treatment of cancer, strokes
- Today: Calculated voxel by voxel
- Here: Model blood flow and use data assimilation

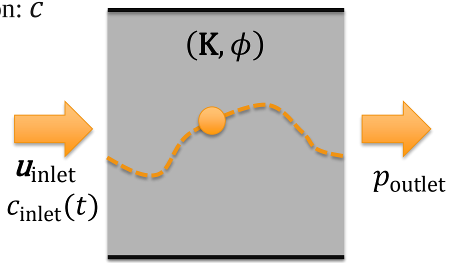
Flow of blood and contrast indicator

Incompressible porous media: $\boldsymbol{u} = -\mu^{-1}\mathbf{K} \cdot \nabla p$, $\nabla \cdot \boldsymbol{u} = 0$

Passive tracer: $\phi \frac{\partial c}{\partial t} - \nabla \cdot (c\boldsymbol{u}) = 0$

- Contrast indicator concentration: c
- Filtration velocity: $\boldsymbol{u}$
- Pressure: p

- Blood viscosity: μ
- Flow conductivity: $\mu^{-1}\mathbf{K}$
- Porosity: ϕ



Two-compartment model ("dual por. / dual perm.")

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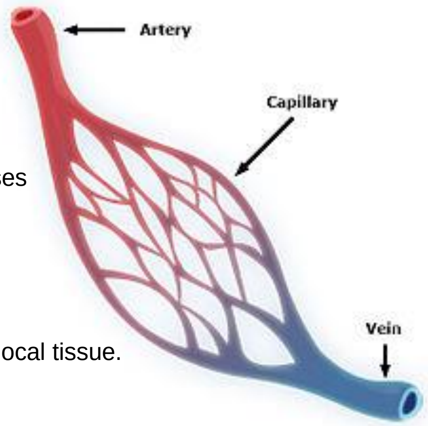
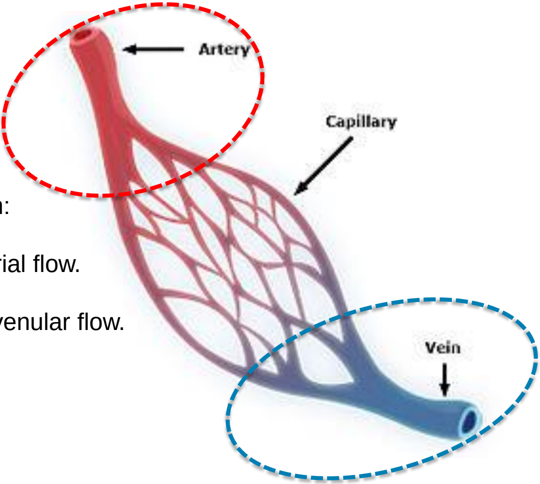


The amount of constrast indicator observed within a «voxel» comprises

- Arterial transit.
- Venular transit.
- Capillary perfusion feeding the local tissue.

(https://en.wikipedia.org/wiki/Capillary)

Two-compartment model (“dual por. / dual perm.”)



Decompose the flow problem:

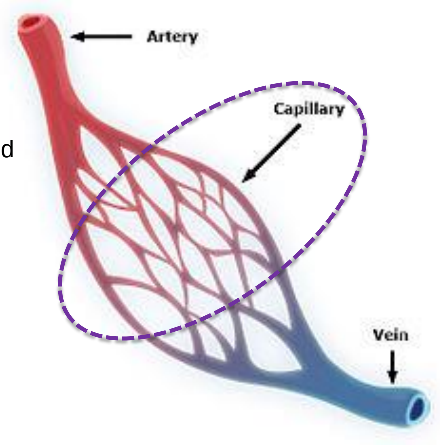
- One <u>global</u> model for arterial flow.
- Another <u>global</u> model for venular flow.
- …

Two-compartment model ("dual por. / dual perm.")

IRIS

Capillary perfusion is represented by <u>local</u> transfer funtions:

- Sinks for the arterial flow.
- Sources for venular flow.



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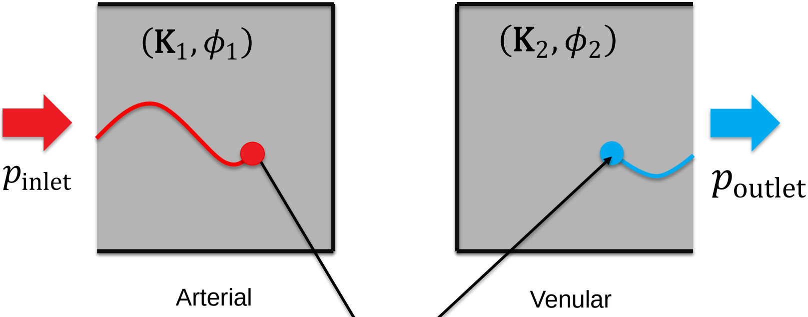

Transfer between arterial and venular system, relates explicitly to perfusion as «feeding blood flow to the tissue».

$$\nabla \cdot \boldsymbol{u}_1 = -Q = -\mu^{-1} \mathrm{K}_X \sigma (p_2 - p_1)$$

$$\nabla \cdot \boldsymbol{u}_2 = Q$$

Ensemble Kalman filter (Evensen (1994))

$x_n = F(x_{n-1}) + \epsilon_n$
$y_n = Gx_n + \eta_n$

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Ensemble Kalman filter solution:

$$X_n = [x_{n,1} \dots x_{n,N}]$$ ensemble of size N

$$x_{n,i} = F(x_{n-1,i}) + \epsilon_{n,i}$$

$$y_{n,i} = Gx_{n,i} + \eta_{n,i}$$ (linear observations)

$$\bar{x}_n = \frac{1}{N}\sum_{i=1}^{N} x_{n,i}$$

$$\bar{C}_n = \frac{1}{N-1}\sum_{i=1}^{N}(x_{n,i} - \bar{x}_n)(x_{n,i} - \bar{x}_n)^T$$

$$K_n = \bar{C}_n G^T (G\bar{C}_n G^T + C_D)^{-1}$$ Estimated Kalman gain

$$\hat{x}_{n,i} = x_{n,i} + K_n(y_{o,n} - y_{n,i})$$ $i = 1, \dots, N$

Parameter estimation using EnKF

$$\begin{bmatrix} x_n \\ p_n \\ y_n \end{bmatrix} = \begin{bmatrix} F(x_{n-1}) \\ p_{n-1} \\ G(x_n) \end{bmatrix}$$

p_n:	unknown parameter vector (porosity, tissue permeability)
x_n:	pressure, saturations
$y_n = G(x_n)$:	non-linear measurements

Lorentzen et. al. 2001 (parameter estimation)
Nævdal et. al. 2002 (+ non-linear measurements)

Implementation for presented cases

Here: Half-iterative EnKF (or modification)

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Case 1:

> Adaptive Gaussian mixture filter
  - Stordal et. al., Computational Geosciences, Vol. 15, 2011

> 500 model realizations

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Case 1:

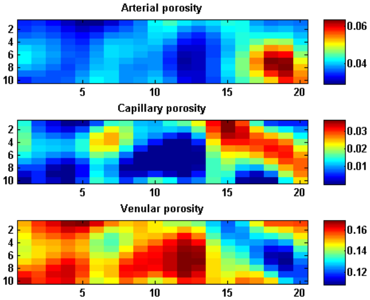
- Adaptive Gaussian mixture filter
  - Stordal et. al., Computational Geosciences, Vol. 15, 2011
- 500 model realizations

Case 2:

- EnKF with extreme localization
- Only updating grid block where contrast is measured

Case study 1

- $10 \text{ cm} \times 10 \text{ cm}$
- Upper, lower and right side have zero-flow boundary conditions
- Pressure at boundary:
  Arterial: 13300 Pa
  Venular: 133 Pa
- Arterial inlet: Pulse lasting 4 seconds, $c_a = 1$
- Contrast agent concentration: $m_{mri}(\vec{x}) = \phi_a \cdot c_a + \phi_{av} \cdot c_{av} + \phi_v \cdot c_v$



Porosity values

- Measurement every 0.6sec
- Measurement noise: 10^{-4}

Estimating perfusion

- Darcy law: $\vec{u}_i = -\mu^{-1} K_i(\vec{x}) \nabla p_i$
- Estimate $K_i(\vec{x})$
- $K_a(\vec{x}) = K_v(\vec{x})$
- Conductivity factor $K_{av}(\vec{x}) \sigma_{av}(\vec{x}) \sim 10 K_a(\vec{x})$

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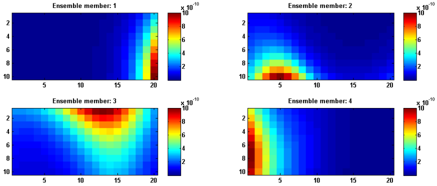
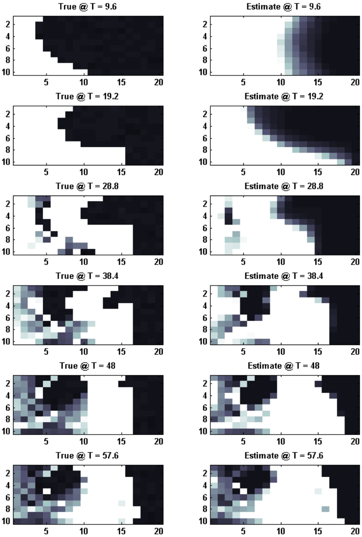


K_a

Four initial ensemble members

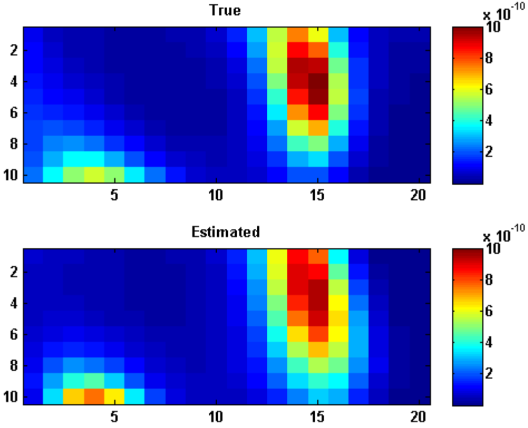
The units are [m^2].

Observed and simulated MRI data



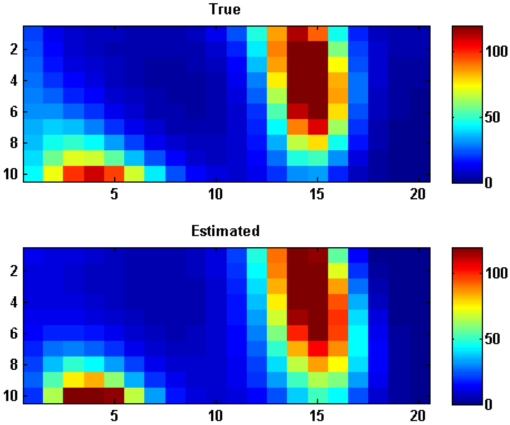
- Black: Low concentration
- White: High concentration
- Range: $[0, 10^{-2}]$.

Estimated K_a field



Units [m^2].

Estimated perfusion

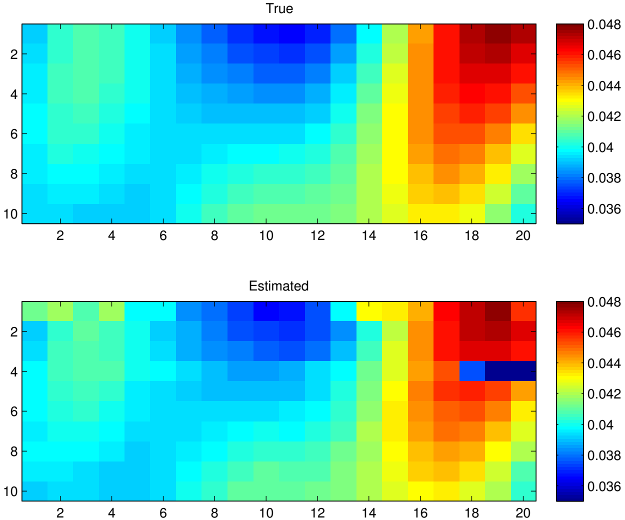


mL/min/100mL

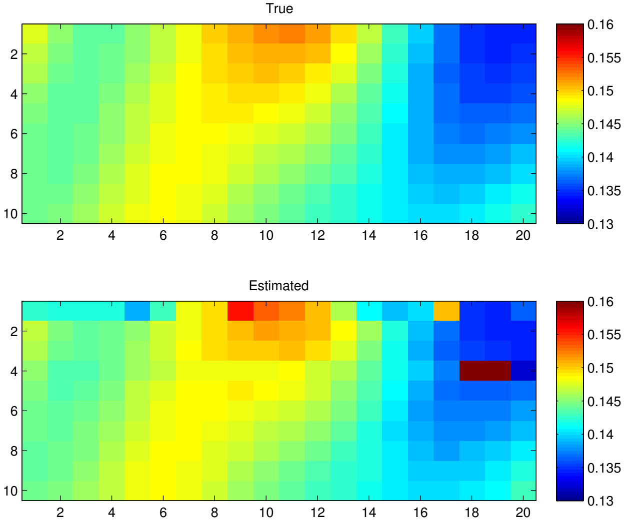
Case 2: Porosity of compartments

- Goal: Estimate $\phi_a(\vec{x})$, $\phi_v(\vec{x})$ and $\phi_{av}(\vec{x})$
- Constant hydraulic conductivity:
  - $K_{av} = 2 \cdot 10^{-9}\mathrm{m}^{-2}$
  - $K_a = K_v = 2 \cdot 10^{-10}\mathrm{m}^{-2}$
- Initial ensemble:
  - geostatistical distribution of the fields
  - $\phi_v + \phi_a + \phi_{av} = 0.2$
- EnKF, ensemble size 200
- Localization: Only update porosities based on observations in its own grid block

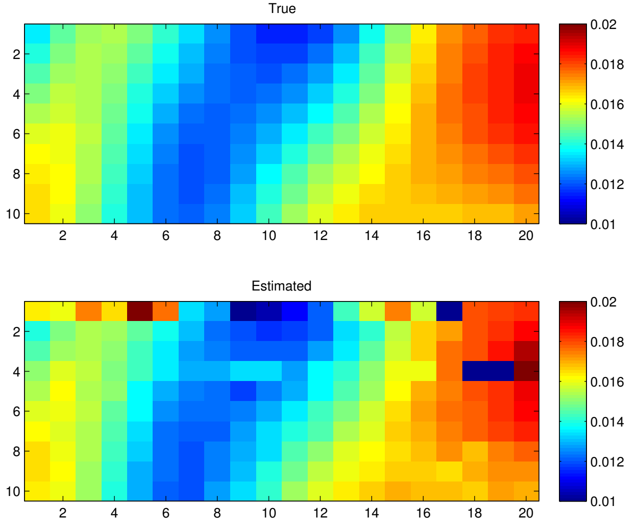
True and estimated ϕ_a



True and estimated ϕ_V



True and estimated ϕ_{av}



Conclusion

- Paper available at
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Thank you!