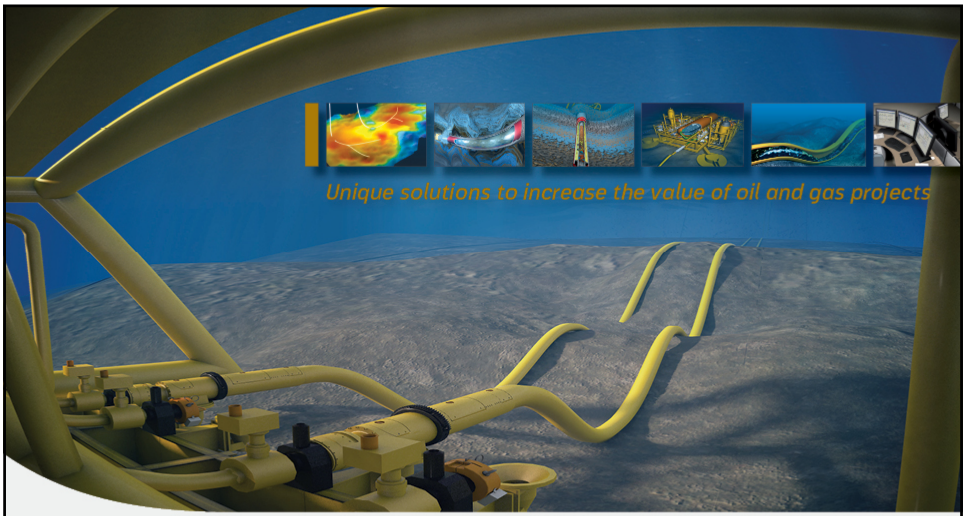

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Stochastic [Spectral] Methods in the Context of Hydrocarbon Reservoir History Matching

Oliver Pajonk[1,2]

[1]SPT Group GmbH, Hamburg, Germany
[2]Institute of Scientific Computing, TU Braunschweig, Germany

be dynamic®

Outline

- Motivation
- Stochastic Methods for Uncertainty Quantification
  - Stochastic Spectral Proxy Models
  - Outlook: Inversion Methods based on Proxies
- Numerical Example
  - Building a Stochastic Spectral Proxy Model for Reservoir Simulation
- Discussion

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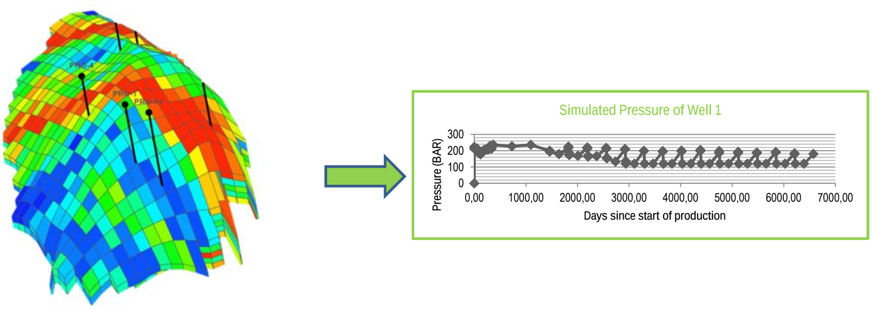
Uncertainty Quantification (UQ): Forward Problem



Task: solve $\boldsymbol{y}_t = h(\boldsymbol{x}_t)$ via simulation; $\boldsymbol{x}_t$ is uncertain – how does that influence the output?
Difficulties: many uncertain parameters; simulation expensive; propagation should be exact, but typically cannot modify simulation code

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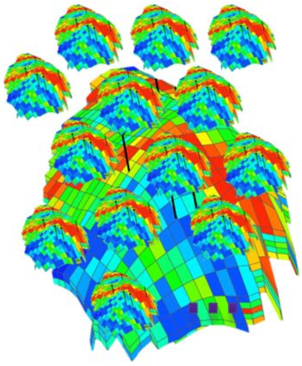
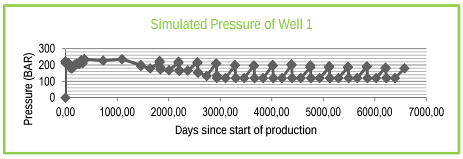
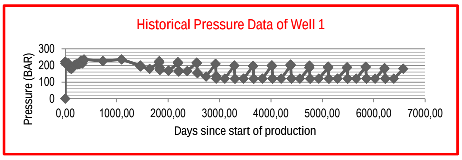
Uncertainty Quantification (UQ): Inverse Problem





Historical data z_t: assume that $z_t = h(\breve{x}_t) + \epsilon$

- $\breve{x}_t$ represents the "true" state and parameters (unknown)



Task: What does uncertain data z_t tell about uncertain input x_t?

Difficulties: h *not invertible*; historical data noisy; ill-posed problem, not uniquely solvable.

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Stochastic Methods for Uncertainty Quantification
Basics & Notation

1. Introduce a parameter $\boldsymbol{\omega}$ describing uncertainty,
2. Use probability theory to quantify it.

- Primary quantities: **random variables** (RVs; here: of finite variance): $\boldsymbol{x}(\omega), \boldsymbol{y}(\omega), \boldsymbol{z}(\omega), \boldsymbol{\epsilon}(\omega) \in L_2(\Omega; V)$
  - Ω: sample space of possible outcomes, V: vector space.
- **Inherent treatment of uncertainties** from different sources
  - Uncertain initial state & parameters; model uncertainties; measurement noise
- Inverse problem **no longer ill-posed**
- **Inference:** Bayes's rule → conditional expectation (CE)
  - Consistent way to include new information (more on that later)

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Stochastic Methods for Uncertainty Quantification
Computer Representation of Random Variables

- Well known: (Monte Carlo) **sampling representation:**

$$R = \{r_i\}, \qquad i \in [1, N], N \gg 1,$$
$$r_i = r(\omega_i), \qquad \omega_i \sim P$$

  - MC sampling + LCE
    → Ensemble Kalman Filter (EnKF) and related methods
  - Known advantages and drawbacks. Can we do better?
- Another popular possibility: **spectral representation:**

$$r(\omega) = \sum_{\alpha \in J} r^{\alpha} f_{\alpha}(\xi_1(\omega), \xi_2(\omega), \dots)$$

  - Series of known functions and basis RVs; spectral coefficients
  - Good: Fast convergence, no random sampling

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Stochastic Methods for Uncertainty Quantification
Polynomial Chaos Expansion – A Stochastic Spectral Proxy Model

- Wiener's Polynomial Chaos Expansion (PCE) using Hermite polynomials:

$$\boldsymbol{r}(\omega) = \sum_{\alpha \in J} \boldsymbol{r}^{\alpha} H_{\alpha}(\theta_1(\omega), \dots, \theta_k(\omega), \dots)$$

- Orthogonal basis functions, standard normal basis RVs
- Others are known and possible, e.g.:
  - Wiener-Askey: Legendre + Uniform, Jacobi + Beta, ….
  - "arbitrary" PC: construct from data

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Stochastic Methods for Uncertainty Quantification
Polynomial Chaos Expansion – A Stochastic Spectral Proxy Model

- Question: How to efficiently compute coefficients $\boldsymbol{r}^{\alpha}$?
- Approach 1: "Intrusive" method
  - Implement constitutive law based on spectral expansion
  - Results in large coupled systems of equations
  - Often infeasible: no access to code, too difficult / costly to change code
- Approach 2: Orthogonality → Use projection:

$$\forall \alpha: \; r^{\alpha} = \langle r | H_{\alpha} \rangle / \langle H_{\alpha} | H_{\alpha} \rangle$$

  - Needs high-dimensional "integrals" (interpolation) over Ω
  - One way: Collocation
    - Interpolation-rules based on polynomial basis, e.g. Gauss-Hermite
  - Full tensor grid not feasible → Use "Smolyak sparse grids"

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Stochastic Methods for Uncertainty Quantification
Bayesian Inversion / Conditioning

- Classical tool of inference: **Bayes's theorem** gives conditional probability measure of "model given data".
  - Use MCMC + stochastic proxy to compute posterior

- More "modern", equivalent: **Conditional expectation** (CE) computes *expectation* with this posterior measure.
- Inverse problem becomes: Compute $\hat{\boldsymbol{x}} = E(\boldsymbol{x}|\boldsymbol{x}, \boldsymbol{z})$
- CE defined as orthogonal projection (L_2 → Hilbert space) of $\boldsymbol{x}$ ("prior") on the *subspace generated by all measurable functions* of $\boldsymbol{x}$ and $\boldsymbol{z}$:
  - $span\{\breve{\boldsymbol{x}}\ |\breve{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{z})$ for some $f\}$.

- $\hat{\boldsymbol{x}}$ ("posterior") optimal in the mean square sense
  - Very direct approach, no sampling
  - Affine approximation → similar to EnKF; square root approach exists
  - Iterative / non-linear extensions topic of current research

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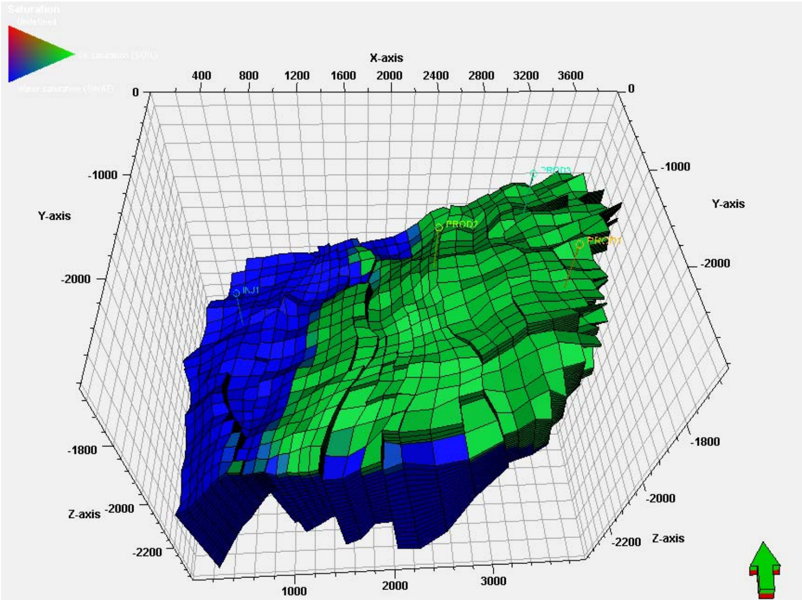
Numerical Example
Building a Stochastic Proxy Model for Reservoir Simulation



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Numerical Example
Building a Stochastic Proxy Model for Reservoir Simulation

- Grid: $31 \times 21 \times 17 = 11067$ cells, 9955 active
- Water-oil system
- 14 faults, three main sand bodies (layers 1-6, 7-12, 13-17)
- One aquifer in central north, connected to lowest sand body
- Three producers, one injector

- Nine independent uncertain parameters:
  - Four main fault multipliers
  - Three permeability multipliers
  - Two z-transmissibility multipliers (layers 6, 12)

- A priori determined "reasonable" parameter values using optimization
- Then: Consider each parameter φ as Gaussian RV with $p\%$ std. dev., i.e.

$$\varphi(\omega) \sim N(\varphi_0, p/100\ abs(\varphi_0))$$

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Numerical Example
Building a Stochastic Proxy Model for Reservoir Simulation

- Task: Proxy model for *field oil production total* (FOPT) after 6 years
  - Note: Building additional proxies is very cheap once collocation points are known!
  - Input uncertainty considered: 5%, unless stated otherwise

- Methods:
  - Build PCE proxy of maximum polynomial order 3, using:
  1. Full tensor grid of Gauss-Hermite points
     - Requires $3^9 = 19683$ simulations
  2. Smolyak sparse grid of Gauss-Hermite points
     - Requires 181 simulations

  - Each proxy has 220 coefficients
  - For comparison: MCMC sampling with 50000 samples

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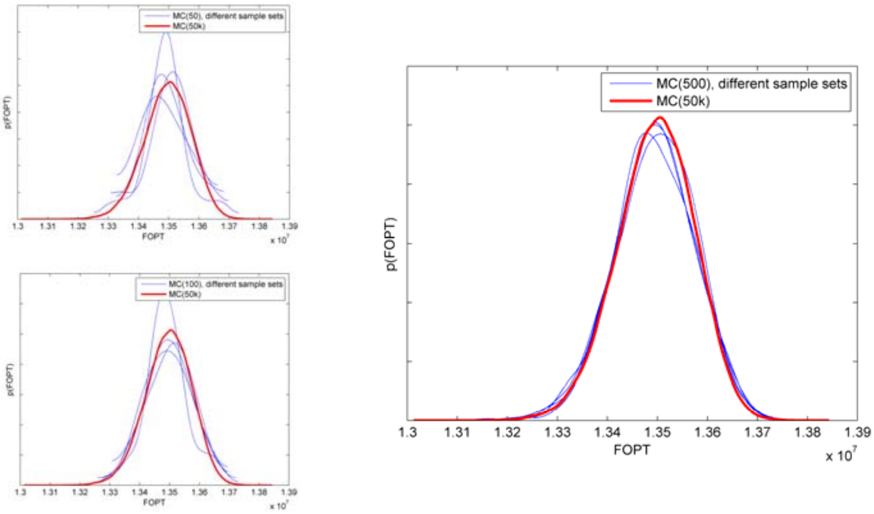
Numerical Example
Before: Monte Carlo – A Word of Warning

- Convergence of Monte Carlo is slow (of course... just as reminder ☺)



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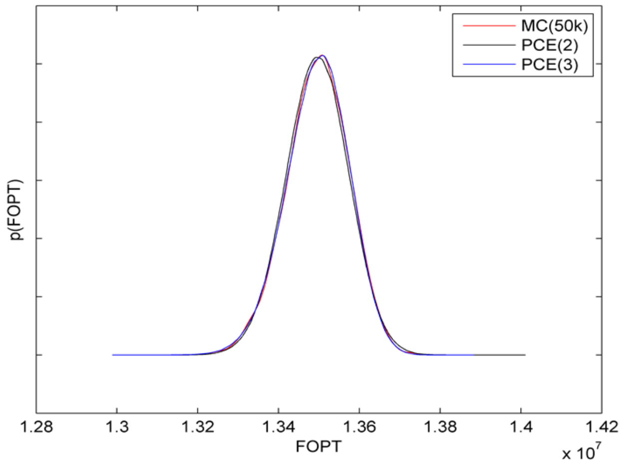
Numerical Example
Results: Full Tensor Grid, PCE of Orders 2 and 3



– PCE(2) is *slightly* off, PCE(3) has converged to MC result
– But 19683 simulations are obviously a problem ☺

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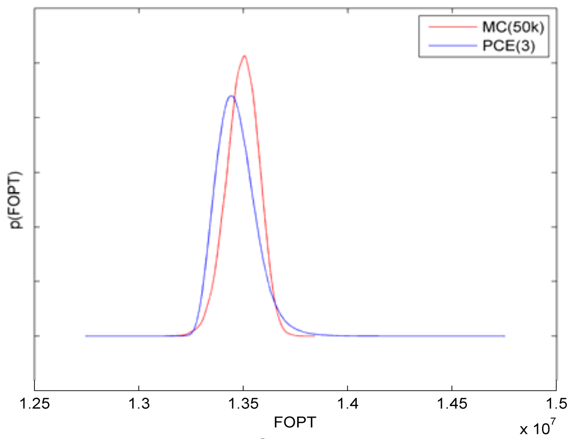
Numerical Example
Results: Smolyak Sparse Grid, PCE of Order 3



– Ouch… that does not work ☹
– An important lesson for Smolyak grids:
  Smolyak has negative integration weights - your integrand should not be "noisy"!
– Here: Adaptive time-stepping (!) and (likely) also solution precision are a problem (under further investigation…)

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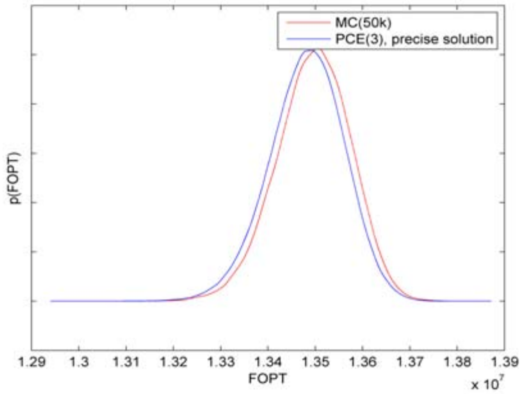
Numerical Example
Results: Smolyak Sparse Grid, PCE of Order 3, “Precise” Simulation Results



- Modified simulation time-stepping & solution precision
- Each simulation is obviously slower – but it’s “just” 181 of them!
- Systematic error likely due to differences in precision & stepping – so PCE(3) solution may be even *better* than MC solution

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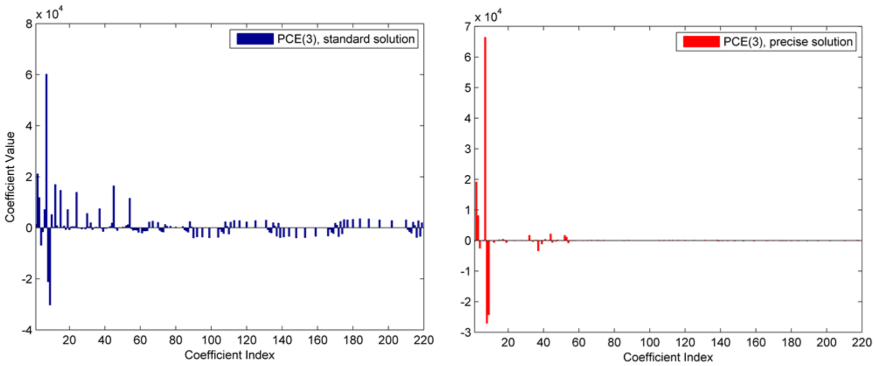
Numerical Example
Results: PCE Coefficients



- First coefficient left out (expected value; very large)
- Both coefficient sets represent same proxy
- One expects that coefficients decrease (due to index ordering by “total degree” of polynomial)
- **Left:** not converged properly, **Right:** converged, many higher terms zero

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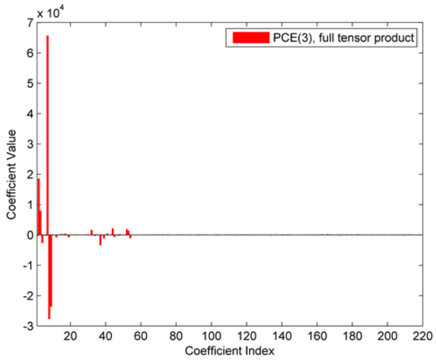
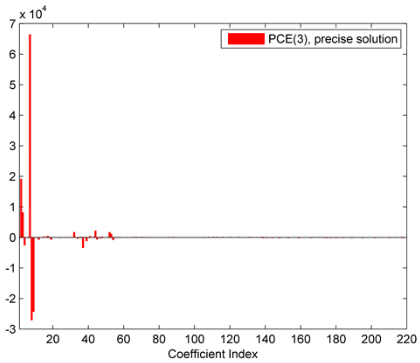
Numerical Example

Results: PCE Coefficients





- First coefficient left out (expected value; very large)
- Both coefficient sets represent same proxy
- **Left:** constructed from full tensor product, **Right:** sparse tensor product
- No visible differences between full tensor and sparse grid

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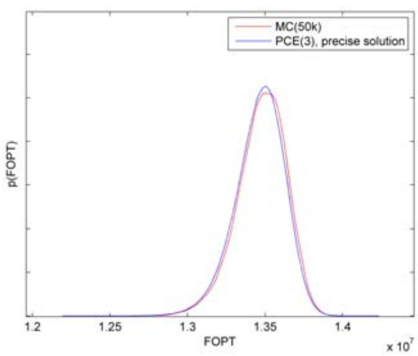
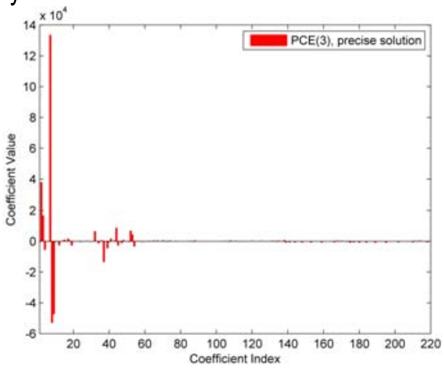
Numerical Example

Results: Similar for 10% Input Uncertainty





- First coefficient left out (expected value; very large)
- Reasonable agreement between MC, PCE
- Differences likely again due to differences in model precision
- Higher-order coefficients become (relative to lower order coefficients) more important – as one would expect, given larger input uncertainty

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Discussion

- PCE is a specific stochastic spectral proxy model
  - PCE just one example; generalisations exist (other distributions)
  - Smolyak quadrature is capable of creating this proxy – but it has certain requirements
  - The approach is applicable to hydrocarbon reservoir simulation
- Demonstration highlighted advantages of spectral representation
  - Better representation of higher moments due to convergence properties
  - Any proxy is very cheap to compute once collocation points are available
  - Use proxy to precisely & rigorously quantify prediction uncertainty
- Use Bayesian updating for history matching (not demonstrated here)
  - Possible to update this proxy *directly* in the Bayesian sense (no sampling, linear approximations are computationally cheap, cf. EnKF)
  - Iterative & non-linear updates topic of research
  - Already possible: Use classical approaches like MCMC to compute update – sampling the proxy is *very* cheap & still precise!

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Discussion

- Difficulties arise with larger numbers of uncertain input parameters (e.g. uncertain property maps)
  - Requires parameter reduction techniques like KLE, PCA, Kernel-PCA, adaptive subspace-techniques, …
- Tuning solver so that solution is “precise enough” for Smolyak may not be simple – but probably worth it

Acknowledgements

- Functions related to Hermite basis are from SGLib (https://github.com/ezander/sglib)

Some Selected References

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