## Recent Advances in EnKF: Running in Place, Assimilation of Rain, Ens. Forecast Sensitivity

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# Promising new tools for the LETKF (1)

**1. Running in Place** (Kalnay and Yang, QJ 2010, Yang, Kalnay and Hunt, MWR, 2012)

 It extracts more information from observations by using them more than once.

- It uses the "no-cost smoother", Kalnay et al., Tellus, 2007b.
- Useful during spin-up (e.g., hurricanes and tornados), and during nonlinear regimes.
- Typhoon Sinlaku (Yang et al., 2012)
- 7-years of Ocean Reanalysis (Penny, 2011, Penny et al., 2012)

# Promising new tools for the LETKF (2)

- **2. Effective assimilation of Precipitation** (Guo-Yuan Lien, Eugenia Kalnay and Takemasa Miyoshi, 2013)
- Assimilation of precipitation has generally failed to improve forecasts beyond a few hours.
- A new approach deals with non-Gaussianity, and assimilation of both zero and non-zero precipitation.
- In OSSEs the model now "remembers" the assimilation, so that that medium range forecasts are improved.
- We are starting to work with real observations... harder...

Promising new tools for the LETKF(3)

3. Forecast Sensitivity to Observations and "proactive QC"

(with Y Ota, T Miyoshi, J Liu, and J Derber)

- A simpler, more accurate formulation for the Ensemble Forecast Sensitivity to Observations (EFSO, Kalnay et al., 2012, Tellus).
- Ota et al., 2012 tested it with the NCEP EnSRF-GFS operational system using <u>all operational observations</u>.
- Allows to identify "bad observations" after 12 or 24hr, and then repeat the data assimilation without them: "proactive QC".

#### 4. Ensemble Sensitivity: application to EnKF

(with Yang)

• We are testing the use of Singular Vectors based on Ensembles in data assimilation.

# Local Ensemble Transform Kalman Filter (Ott et al, 2004, Hunt et al, 2004, 2007) (a square root filter)

(Start with initial ensemble)



- Model independent (black box)
- Obs. assimilated simultaneously at each grid point
- 100% parallel
- No adjoint needed
- 4D LETKF extension
- Computes the **weights** for the ensemble forecasts explicitly

#### Localization based on observations

Perform data assimilation in a local volume, choosing observations

The state estimate is updated at the central grid red dot



### Localization based on observations

Perform data assimilation in a local volume, choosing observations

The state estimate is updated at the central grid red dot

All observations (purple diamonds) within the local region are assimilated



The LETKF algorithm can be described in a single slide!

#### Local Ensemble Transform Kalman Filter (LETKF)

Globally: Forecast step:  $\mathbf{x}_{n,k}^{b} = M_{n}\left(\mathbf{x}_{n-1,k}^{a}\right)$ Analysis step: construct  $\mathbf{X}^{b} = \left[\mathbf{x}_{1}^{b} - \overline{\mathbf{x}}^{b} \mid ... \mid \mathbf{x}_{K}^{b} - \overline{\mathbf{x}}^{b}\right];$  $\mathbf{y}_{i}^{b} = H(\mathbf{x}_{i}^{b}); \mathbf{Y}_{n}^{b} = \left[\mathbf{y}_{1}^{b} - \overline{\mathbf{y}}^{b} \mid ... \mid \mathbf{y}_{K}^{b} - \overline{\mathbf{y}}^{b}\right]$ 

Locally: Choose for each grid point the observations to be used, and compute the local analysis error covariance and perturbations in ensemble space:

$$\tilde{\mathbf{P}}^{a} = \left[ \left( K - 1 \right) \mathbf{I} + \mathbf{Y}^{\mathbb{F}} \mathbf{R}^{-1} \mathbf{Y}^{b} \right]^{-1}; \mathbf{W}^{a} = \left[ (K - 1) \tilde{\mathbf{P}}^{a} \right]^{1/2}$$

$$= a \tilde{\mathbf{p}}^{a} \mathbf{Y}^{bT} \mathbf{p}^{-1} (a - b)$$

Analysis mean in ensemble space:  $\mathbf{\bar{w}}^{a} = \mathbf{P}^{a}\mathbf{Y}^{bT}\mathbf{R}^{-1}(\mathbf{y}^{o}-\mathbf{\bar{y}}^{b})$ and add to  $\mathbf{W}^{a}$  to get the analysis ensemble in ensemble space.

The new ensemble analyses in model space are the columns of  $\mathbf{X}_{n}^{a} = \mathbf{X}_{n}^{b}\mathbf{W}^{a} + \overline{\mathbf{x}}^{b}$ . Gathering the grid point analyses forms the new global analyses. Note that the the output of the LETKF are analysis weights  $\overline{\mathbf{w}}^{a}$  and perturbation analysis matrices of weights  $\mathbf{W}^{a}$ . These weights multiply the ensemble forecasts.

**No-cost LETKF smoother** ( $\times$ ): apply at t<sub>n-1</sub> the same weights found optimal at t<sub>n</sub>. It works for 3D- or 4D-LETKF



The no-cost smoother makes possible:

- ✓ Quasi Outer Loop (QOL)
- ✓ "Running in place" (RIP) for faster spin-up
- ✓ Use of future data in reanalysis
- ✓ Ability to use longer windows and nonlinear perturbations

# No-cost LETKF smoother first tested on a QG model: it works...



Very simple smoother: apply the final weights at the beginning of the window. It allows assimilation of **future** data, and assimilating data more than once.<sup>10</sup>

#### Nonlinearities: "Quasi Outer Loop" (QOL)

Quasi Outer Loop: use the final weights to correct only the <u>mean</u> initial analysis, keeping the initial perturbations. Repeat the analysis once or twice. It re-centers the ensemble on a more accurate nonlinear solution.



#### Nonlinearities, "QOL" and "Running in Place"

Quasi Outer Loop: similar to 4D-Var: use the final weights to correct only the <u>mean</u> initial analysis, keeping the initial perturbations. Repeat the analysis once or twice. It centers the ensemble on a more accurate nonlinear solution.

Lorenz -3 variable model RMS analysis error

	4D-Var	LETKF	LETKF	LETKF
			+QOL	+RIP
Window=8 steps	0.31	0.30	0.27	0.27
Window=25 steps	0.53	0.68	0.47	0.35

"Running in Place" smoothes both the analysis and the analysis error covariance and iterates a few times...

# Running in Place: Spin-up with a QG model



RIP accelerates the EnKF spin-up (e.g., hurricanes, severe storms)

Spin-up depends on the initial perturbations, but RIP works well even with uniform random perturbations. RIP becomes even faster than 4D-Var (blue).

### Why RIP works: Results with a Linear model



- RIP adapts to using an observation N-times by dividing the spread by N: RIP converges to the regular optimal KF solution.
- The spin-up is faster and the analysis update is "softer" (in small steps) rather than in large steps.

# LETKF-RIP with real observations (Typhoon Sinlaku, 2008)



11/23/2011@NTU-TIMS

#### **Observation Impact for the first set of dropsondes**



The effectiveness of the dropsonde data is greatly improved by RIP and the negative impact shown in the control LETKF is much reduced.

An application of LETKF-RIP to ocean data assimilation

# Data Assimilation of the Global Ocean using 4D-LETKF, SODA(OI) and MOM2

Steve Penny's thesis defense April 15, 2011

Advisors: E Kalnay, J Carton, K Ide, T Miyoshi, G Chepurin

Penny (now at UMD/NCEP) implemented the LETKF with either IAU or RIP and compared it with SODA (OI)



Global RMS(O-F) of Temperature (°C), 12-month moving average LETKF (with IAU), SODA and LETKF with RIP<sup>18</sup>

#### RMSD (psu) (All vertical levels)

B: background A: analysis



Why is LETKF-RIP so much better than SODA or LETKF-IAU for the ocean reanalysis?

- The ocean observations are too sparse for a standard EnKF, or even OI/3D-Var with a short (5day) window.
- SODA and LETKF-IAU used a much longer window (30 days) in order to hammer the system with the available observations.
- LETKF-RIP is able to use a 5-day window but reuses the observations in order to extract more information.

# Summary for LETKF-RIP (or QOL)

- Kalman Filter is optimal for a linear, perfect model.
- During spin-up, or when the ensemble perturbations grow nonlinearly, EnKF is not optimal, since it cannot extract enough information from the observations.
- The LETKF "no-cost" smoother (or, equivalently, the 4D-EnSRF) allows LETKF-RIP to use the observations more than once, and thus extract more information.
- This shortens the spin-up and produces more accurate forecasts with the same observations.
- For linear models RIP converges to the same optimal KF solution but with spread reduced by  $\sim \sqrt{N}$
- For long windows and nonlinear perturbations, RIP advances in smaller steps and approaches the true attractor more "21 "softly".

### (2) Effective Assimilation of Precipitation

(Guo-Yuan Lien, E. Kalnay and T Miyoshi)

- Assimilation of precipitation has been done by changing the moisture Q in order to make the model "rain as observed".
- Successful during the assimilation: e.g. the North American Regional Reanalysis had perfect precipitation!
- However the model forgets about the changes soon after the assimilation stops!
- The model **will remember** potential vorticity (PV).
- EnKF should modify PV efficiently, since the analysis weights will be larger for an ensemble member that is raining more correctly, because it has a better PV.
- However, 5 years ago, we had tried assimilating precipitation observations in a LETKF-SPEEDY OSSE but the results were POOR!
- Problem: precipitation is very non-Gaussian.
- We tried a Gaussian transformation of precipitation and it worked!

#### How do we transform precipitation y to a Gaussian $y_{transf}$ ?

Start with pdf of y=rain at every grid point.

"No rain" is like a delta function that we cannot transform.

We assign all "no rain" to the **median** of the no rain CDF.

We found this works as well as more complicated procedures.

It allows to assimilate both rain and no rain.





- Main result: with at least 10 ensemble members raining in order to assimilate an obs, updating all variables (including vorticity), with Gaussian transform, and rather accurate observations (20% errors), the analyses and forecasts are much improved!
- Updating only Q is much less effective.
- The 5-day forecasts maintain the advantage.





The model remembers the impact of pp assimilation in the SH, NH and tropics!



If we assimilate only rain the results are much worse! We need to assimilate both rain and no rain!



The impact of the Gaussian Transform is important with large observation errors (50% rather than 20%). The impact of GT50% is almost as good as GT20%.

#### Vorticity errors and corrections



There is no vorticity information in the pp observations, but the LETKF clearly knows about the vorticity errors <sup>28</sup>

#### How about real observations? We will use TRMM/TMPA satellite estimates (from G. Huffman) with the NCEP GFS

TRMM 3B42 Zero-Prcp Probability (%) [All Seasons]



TRMM/TMPA: 14 years of data, 50S-50N, 3hrs, 0.5 deg

# Summary for assimilation of precipitation

- The model <u>remembers potential vorticity (dynamics)</u>, <u>does not</u> <u>remember moisture changes</u>, or even temperature.
- For this reason, when using nudging, or variational assimilation of precipitation to change Q and T, the model "forgets" this information and returns to the original forecast.
- EnKF has a better chance to assimilate <u>potential vorticity</u> by giving higher weights to ensemble members with good precip.
- In addition, EnKF has the advantage of not requiring model linearization, a problem for variational systems.
- In OSSEs EnKF with a Gaussian transformation of precipitation assimilates rain info and remembers it during the forecast.
- Requiring at least several ensemble forecasts to have Rain>0 allows the effective assimilation of both <u>rain</u> and <u>no rain</u>.

#### The NCEP 5-day skill dropout problem



## **Ensemble Forecast Sensitivity to Observations**

"Adjoint sensitivity without adjoint" (Liu and K, 2008, Li et al., 2010) We now have a simpler, more accurate formulation

(Kalnay, Ota, Miyoshi: Tellus, 2012)



 $\mathbf{e}_{t|0} = \overline{\mathbf{x}}_{t|0}^{f} - \overline{\mathbf{x}}_{t}^{a}$ 

(Adapted from Langland and Baker, 2004)

The only difference between  $\mathbf{e}_{t|0}$  and  $\mathbf{e}_{t|-6}$  is the assimilation of observations at 00hr:

$$(\overline{\mathbf{x}}_{0}^{a} - \overline{\mathbf{x}}_{0|-6}^{b}) = \mathbf{K}(\mathbf{y} - H(\mathbf{x}_{0|-6}^{b}))$$

> Observation impact on the reduction of forecast error:

$$\Delta \mathbf{e}^{2} = (\mathbf{e}_{t|0}^{T} \mathbf{e}_{t|0} - \mathbf{e}_{t|-6}^{T} \mathbf{e}_{t|-6}) = (\mathbf{e}_{t|0}^{T} - \mathbf{e}_{t|-6}^{T})(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})$$

#### **Ensemble Forecast Sensitivity to Observations**

$$\Delta \mathbf{e}^{2} = (\mathbf{e}_{t|0}^{T} \mathbf{e}_{t|0} - \mathbf{e}_{t|-6}^{T} \mathbf{e}_{t|-6}) = (\mathbf{e}_{t|0}^{T} - \mathbf{e}_{t|-6}^{T})(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})$$
$$= (\overline{\mathbf{x}}_{t|0}^{f} - \overline{\mathbf{x}}_{t|-6}^{f})^{T}(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})$$
$$= \left[\mathbf{M}(\overline{\mathbf{x}}_{0}^{a} - \overline{\mathbf{x}}_{0|-6}^{b})\right]^{T}(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}), \text{ so that}$$
$$\Delta \mathbf{e}^{2} = \left[\mathbf{M}\mathbf{K}(\mathbf{y} - H(\mathbf{x}_{0|-6}^{b}))\right]^{T}(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})$$

Langland and Baker (2004), Gelaro, solve this with the adjoint:

$$\Delta \mathbf{e}^2 = \left[ (\mathbf{y} - H(\mathbf{x}_{0|-6}^b)) \right]^T \mathbf{K}^T \mathbf{M}^T (\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})$$

This requires the adjoint of the model  $\mathbf{M}^{T}$  and of the data assimilation system  $\mathbf{K}^{T}$  (Langland and Baker, 2004)

#### **Ensemble Forecast Sensitivity to Observations**

Langland and Baker (2004):

$$\Delta \mathbf{e}^{2} = \left[ \mathbf{M} \mathbf{K} (\mathbf{y} - H(\mathbf{x}_{0|-6}^{b})) \right]^{T} (\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})$$
$$= \left[ (\mathbf{y} - H(\mathbf{x}_{0|-6}^{b})) \right]^{T} \mathbf{K}^{T} \mathbf{M}^{T} (\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})$$

With EnKF we can use the original equation without "adjointing": Recall that  $\mathbf{K} = \mathbf{P}^{a}\mathbf{H}^{T}\mathbf{R}^{-1} = 1 / (K-1)\mathbf{X}^{a}\mathbf{X}^{aT}\mathbf{H}^{T}\mathbf{R}^{-1}$  so that  $\mathbf{M}\mathbf{K} = \mathbf{M}\mathbf{X}^{a}(\mathbf{X}^{aT}\mathbf{H}^{T})\mathbf{R}^{-1} / (K-1) = \mathbf{X}_{t|0}^{f}\mathbf{Y}^{aT}\mathbf{R}^{-1} / (K-1)$ 

Thus,  
$$\Delta \mathbf{e}^{2} = \left[ \mathbf{M} \mathbf{K} (\mathbf{y} - H(\mathbf{x}_{0|-6}^{b})) \right]^{T} (\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})$$
$$= \left[ (\mathbf{y} - H(\mathbf{x}_{0|-6}^{b})) \right]^{T} \mathbf{R}^{-1} \mathbf{Y}_{0}^{a} \mathbf{X}_{t|0}^{fT} (\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}) / (K-1)$$

This uses the available nonlinear forecast ensemble products.

#### Tested ability to detect a poor quality ob impact on the forecast in the Lorenz 40 variable model



Observation impact from LB(+) and from ensemble sensitivity (•)

# Impact of dropsondes on a Typhoon

(Kunii et al. 2012)



#### **Denying negative impact data improves forecast!**



#### Ota et al. 2013, Tellus: Applied EFSO to NCEP GFS/ EnSRF using all operational observations. New: identified regional 24hr "forecast failures"

- Divide the globe into 30x30° regions
- Find all cases where the 24hr regional forecast error is at least 20% larger than the 36hr forecast error verifying at the same time, and
- where the 24hr forecast has errors at least twice the time average.
- Identify the top observation type that has a negative impact on the forecast.
- Found 7 cases of 24hr forecast failures

#### 24-hr forecast error correction (Ota et al.)

- identified 7 cases of large 30°x30° regional errors,

- rerun the forecasts denying bad obs.

- the forecast errors were substantially reduced

- this could be applied to improve the 5-day skill dropouts

	Initial	Area	Size	Rate	Ν	Denied observation	Change	
	12 UTC	90S~60S	2.04	2.04 1.20	1	GPSRO (80S~60S, 90E~120E)	-6.6%	
	JAN 10	100E~130E				ASCAT (60S~50S, 100E~120E)	0.070	
(	06 UTC JAN 12 50N~80N 150E ~ 18		2.18	1.40	1	AMSUA (ch4: 45N~75N,		
		50N~80N 150E ~ 180				$160E \sim 170W$ , ch5:40N $\sim 55N$ ,		
						155E~180, NOAA15 ch6:	-11.4%	
						50N~75N, 140E~170W, ch7:		
						70N~80N, 130E~170E)		
	00 LITC 20N- 60N				Radiosonde wind (Valentia,			
	$\frac{100010}{14}$	30W~0	2.13	1.31	2	Ireland), ASCAT (40N~47N,	-1.0%	
	JAN 10					20W~10W, 50N~55N, 35W~30W)		
		005.605	2.34	1.22	2	AMSUA (ch5: 65S~50S, 90E~110E,		
	JAN 22 J30E~160E	903~003				60S~50S, 120E~127E, ch6:	-2.2%	
		130E~100E				60S~45S, 110E~125E)		
	06 UTC 50N~80N FEB 2 150W~120W	50NL 80N		0 1.32	4	IASI (35N~45N, 155W~150W)		
		150W 120W	3.10			NEXRAD (55N~60N,	-5.5%	
					160W~135W)			
	18 UTC	60N~90N	2.06	1 71	$\mathbf{r}$	MODIS_Wind (60N~90N,	20.00/	
12	FEB 6	50E~80E	2.00			30E~90E)	-39.0%	
V	18 UTC	90S~60S	2 56	1 22	.22 1	MODIS Wind (805, 505, 20W, 0)	22 50/	
	FEB 6	20W~10E	5.50	1.22			-22.370	

N/()

#### "Proactive" QC: Bad observations can be identified by EFSO and withdrawn from the data assimilation



After identifying MODIS polar winds producing bad 24 hr regional forecasts, the withdrawal of these winds reduced the forecast errors by 39%, as projected by EFSO.

# Standard application: Impacts of Observing Systems. EnKF allows using moist total energy



The EnKF formulation is nonlinear and thus allows computing Moist Total Energy and estimate more accurately the impact of the channels on the moisture forecast. Adjoint formulation needs TLM.

#### Another potential application of Ensemble Sensitivity

**5. Application of ensemble forecast sensitivity to data assimilation** (Yang, Kalnay, thanks to Enomoto)

• Very promising!!

Ensemble Sensitivity: Application to Data Assimilation and the Spin-up Problem

Assume we are in a window of the LETKF with an ensemble of K members

$$\mathbf{x}_{i,t}^b = M(\mathbf{x}_{i,t-1}^a)$$

Since the window is short,

$$\delta \mathbf{x}_{i,t}^b = \mathbf{x}_{i,t}^b - \overline{\mathbf{x}}_t^b \approx \mathbf{M}(\delta \mathbf{x}_{i,t-1}^a)$$

Define the vectors of analysis and forecast perturbations:

$$\mathbf{X}_{t-1}^{a} = [\delta \mathbf{x}_{1,t-1}^{a}, \dots, \delta \mathbf{x}_{K,t-1}^{a}]; \quad \mathbf{X}_{t}^{b} = [\delta \mathbf{x}_{1,t}^{b}, \dots, \delta \mathbf{x}_{K,t}^{b}]$$

We want to find the linear combination of analysis perturbations that will grow fastest:

$$\delta \mathbf{x}_{t-1}^{a} = \mathbf{X}_{t-1}^{a} \mathbf{p}; \quad \delta \mathbf{x}_{t}^{b} = \mathbf{X}_{t}^{b} \mathbf{p}$$

with optimal coefficients  $\mathbf{p} = [p_{t,1}, \dots, p_{t,K}]$ 

We can use the equation in Enomoto et al (2007) (see derivation in Yang and Kalnay, 2013):

$$(\mathbf{X}_{t-1}^{aT} C_I \mathbf{X}_{t-1}^{aT})^{-1} (\mathbf{X}_t^{bT} C_F \mathbf{X}_t^{bT}) \mathbf{p} = \lambda \mathbf{p}$$

#### We tested this with a QG model **starting with a random ensemble** that satisfies the B<sub>3D-Var</sub>. The initial optimal perturbation after 6hr grows into a final perturbation after 12 hrs:



Is this fast growing perturbation related to the background errors?

#### We tested this with a QG model **starting with a random ensemble** that satisfies the B<sub>3D-Var</sub>. The initial optimal perturbation after 6hr grows into a final perturbation after 12 hrs:



Is this fast growing perturbation related to the background errors?

#### YES!!!

# Summary

- RIP can extract more information from observations and accelerate spin-up. Examples: Typhoon Sinlaku and Ocean 7 year reanalysis.
- EnKF can be used to assimilate and remember precipitation information, using a Gaussian Transform and other ideas.
- The Ensemble Forecast Sensitivity to Observations can be used to detect observations that result in bad *regional* 12 or 24hr forecasts. This allows repeating analysis without the bad observations: "proactive QC" and monitoring.
- Ensemble Sensitivity may be used to improve the LETKF spinup.
- EnKF is a newer, simpler, powerful technology.

# Extra slides

## How about hybrids between Var and EnKF?

- So far hybrids have been created combining <u>an existing</u> <u>Var system</u> with an ensemble to provide the flow dependence of the background error covariance.
- We would like to start with a well-developed EnKF (like the LETKF) and add a simple local 3D-Var that provides the full rank that the ensemble lacks.
- Steve Penny developed a simple, locally Gaussian 3D-Var for this purpose, and tested it on the Lorenz-96, a 40 variable model.
- He plots the analysis error as a function of the number of ensemble members (2 to 40) and the number of observations (1 to 40).

#### An ensemble based hybrid with a simple local 3D-Var (Steve Penny) applied to the Lorenz 96 model



This is the corner where we are in ocean EnKF: too few obs, too few ensembles

The total model dimension is K=40

The LETKF is extremely accurate as long as k>7, number of obs>7.

#### An ensemble based hybrid with a simple local 3D-Var (Steve Penny) applied to the Lorenz 96 model



The hybrid LETKF-simple 3D-Var is more robust for few ensemble members and few observations, as in the ocean.

#### S. Zhang et al.: GFDL Coupled Ocean-Atm EnKF Data Assimilation



# **Basic idea for our coupled LETKF assimilation**



# Summary: ideas/questions for future coupled ocean-atmosphere EnKF

- Toy model: coupled assimilation and short windows are more accurate for LETKF even if ocean has longer time scales.
- Running in Place (RIP) extracts more information from the observations and allows the use of shorter windows.
- A new hybrid LETKF+simple 3D-Var would make the system more robust with fewer ensemble members and observations.
- For the coupled (India Monsoon Mission) CFS system, we will test the use of 6hr (short) windows for the ocean as well as the atmosphere assimilation.
- Assimilate SST and SSH observations directly.
- Localization of observations near the surface should allow for atm.-ocean interaction through the background error covariance