An adaptive plurigaussian truncation scheme for geological uncertainty quantification using EnKF.

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Outline

Problem

- Let's consider n objects denoted F_1,F_2,. . ., F_n of which probabilities of occurrence in a random experiment are p_1, p_2, . . ., p_n
- Let's sample a set of m these objects based on the given statistic.

A possible solution

- We consider a domain A of measure 1, in a metric space X.
- Split the domain in n sub-domains of which measures are given by the probabilities
- Generate an ensemble of m independent elements in A, having an **uniform** distribution with support on A.
- For each generated element, assign the object in whose subdivision of A belongs

Problem (II)

- Let's consider n objects denoted F_1,F_2,. . ., F_n of which probabilities of occurrence in a random experiment are p_1, p_2, . . ., p_n
- Let's sample a set of m sequences, of k objects, based on the given statistic, with the property that in each sequence some of them are not neighbors.
- Question ? How to solve this problem using Gaussian variables

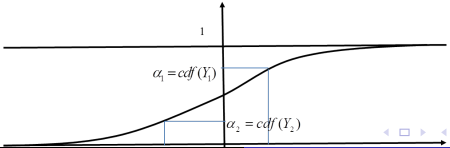
Lemma (I)

Lemma

Let D a sub-domain of the square $[0,1]^2$, and two independent random variables $Y_1 \sim N(0;1)$, $Y_2 \sim N(0;1)$. Then $P((cdf(Y_1), cdf(Y_2)) \in D) = area(D)$.

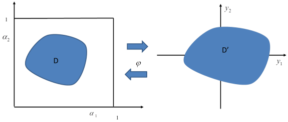
Proof:

$cdf : \mathbf{R} \rightarrow (0,1)$ where, $cdf(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{x^2}{2}} dx$.



Lemma (II)

We define the function $\varphi : \mathbf{R}^2 \to (0,1)^2$,
$\varphi(y_1, y_2) = (cdf(y_1), cdf(y_2))$. Let be $D^{'} = \varphi^{-1}(D)$



Then,

$$area(D) = \iint_D d\alpha_1 d\alpha_2 \qquad (1)$$

Lemma (III)

We perform a change of variables according to $\alpha_1 = cdf(y_1)$ and $\alpha_2 = cdf(y_2)$.

$$\iint_D d\alpha_1 d\alpha_2 = \iint_{D'} |Det(Jac_{(y_1,y_2)}(\alpha_1,\alpha_2))| dy_1 dy_2 \qquad (2)$$

where,

$$Jac_{(y_1,y_2)}(\alpha_1,\alpha_2) = \begin{pmatrix} \frac{\partial cdf(\alpha_1)}{\partial y_1} & \frac{\partial cdf(\alpha_1)}{\partial y_2} \\ \frac{\partial cdf(\alpha_2)}{\partial y_1} & \frac{\partial cdf(\alpha_2)}{\partial y_2} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2\pi}} e^{-\frac{y_1^2}{2}} & 0 \\ 0 & \frac{1}{\sqrt{2\pi}} e^{-\frac{y_2^2}{2}} \end{pmatrix} \qquad (3)$$

Lemma (IV)

Consequently,

$$area(D) = \iint_D d\alpha_1 d\alpha_2 = \frac{1}{2\pi} \iint_{D'} e^{-\frac{y_1^2}{2} - \frac{y_2^2}{2}} dy_1 dy_2 = P((y_1, y_2) \in D^{'}) \quad (4)$$

But, $P((y_1, y_2) \in D^{'}) = P((cdf(y_1), cdf(y_2)) \in D)$ therefore

$$P((cdf(y_1), cdf(y_2) \in D) = area(D) \quad (5)$$

Problem (II)

- Let's consider n objects denoted F_1,F_2,..., F_n of which probabilities of occurrence in a random experiment are p_1, p_2, ..., p_n
- Let's sample a set of m sequences, of k objects, based on the given statistic, with the property that in each sequence some of them are not neighbors.
- Question ? How to solve this problem using Gaussian variables

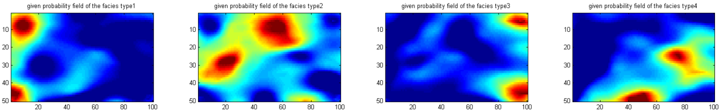
Prior information (I)

1. Number of the facies types that occurs
2. The possible contacts between facies types
3. Facies observations at the well locations (core information)
4. The expected facies proportions (global)

Prior information (II)

Seismic data

For each facies type a probability occurrence map that incorporates the core information.

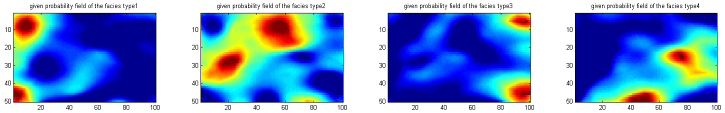


The geological simulation model

- Define a geological simulation model through which we generate facies maps that incorporate the prior information available.
- Condition: For each facies type, the probability map calculated from an ensemble generated with the geological model must resemble with the given probability map

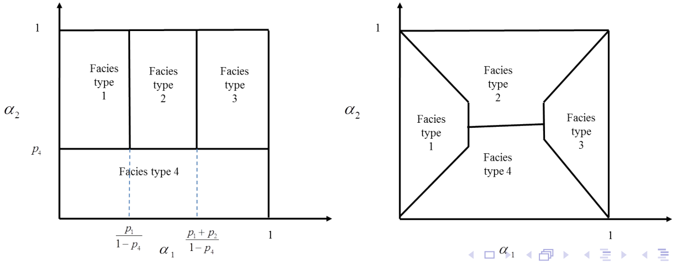
Prior Information: Particular case

1. 4 facies types occurring denoted F_1, F_2, F_3 and F_4
2. The possible contacts between facies types: All possible ,less F_1 with F_3
3. Facies observations at the well locations : yes
4. Facies probability maps for each of them : yes



The simulation maps in probabilities space

- For each grid cell we have four probabilities from the given probabilities maps, denoted p_1, p_2, p_3 and $p_4 = 1 - p_1 - p_2 - p_3$
- Using from the prior information the possible contact between facies types we construct a decomposition of the square $[0, 1]^2$ as:



The facies type simulation at the grid level

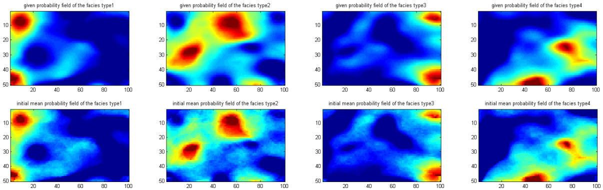
- We generate two independent Gaussian variables y_1 and y_2.
- In the grid cell i we assign the facies type depending on, where the point $(cdf(y_1), cdf(y_2))$ belongs in the simulation map built for the grid i.
- Using lemma, for an ensemble of facies types simulated in this grid cell, the distribution calculated from ensemble is defined by the given probabilities

The facies maps simulation

- We simulate for each grid cell facies types using gaussian random variables
- For the facies continuity in the field we need the spatial correlation of the gaussian variables
- We generate two independent Gaussian random fields Y_1 and Y_2 defined on the reservoir domain.
- In each grid cell i we assign the facies type depending on, where the point $(cdf(Y_1^i), cdf(Y_2^i))$ belongs in the simulation map built for the grid i.

The simulation property

Using Lemma, for an ensemble of facies maps generated, the probability maps calculated from the ensemble resemble with the given probabilities maps.



The adaptivity

- At the well locations, where a facies type is observed, the associated map (in the prob space) is the square $[0, 1]^2$ all occupied with that facies type.
  The consequence: In this grid cell, that facies type is always simulated.
- At the grid cells, where a facies type has the occurrence probability 0, the simulation map does not contain a region assigned to that facies type.
  The consequence: In this grid cell, that facies type is not simulated.

The state vector

The state vector for the j^{th} ensemble member at the k^{th} assimilation step :

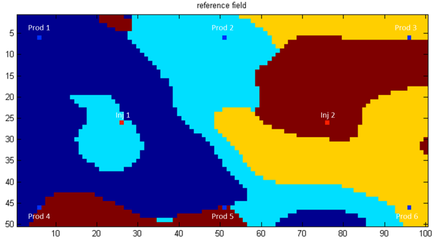
$$\mathbf{x}_j^k = \left[y_1^T \ y_2^T \ \mathbf{d}_{sim}^T\right]_j^\top , \tag{6}$$

where, $\mathbf{d}_{sim}$ are the simulated observations represented by the simulated production data (oil and water rates, bottom hole pressures).

The flow

1. Generate an ensemble of independent Gaussian fields with which we construct the initial ensemble of facies maps.
2. At time step k we assimilate the production data. This affects the Gaussian fields values, which provide a new ensemble of facies maps realizations.
3. At the end of assimilation period we have a geological uncertainty quantification represented by the updated ensemble of facies maps.

Reference field :4 facies types



Facies type	Permeability	Porosity	Colour
Type 1	2 md	0.1	Blue
Type 2	10 md	0.2	Light blue
Type 3	50 md	0.2	Yellow
Type 4	250 md	0.3	Red

Reservoir set up

- 8-spot water flooding 2D-reservoir, black oil model with 100*50*1 active grid blocks.

Table : The position of the wells in the reservoir domain and the facies observations

	Inj 1	Inj 2	Prod 1	Prod 2	Prod 3	Prod 4	Prod 5	Prod 6
x coordinate	25	75	5	50	95	5	50	95
y coordinate	25	25	5	5	5	45	45	45
Facies observation	Type 2	Type 4	Type 1	Type 2	Type 3	Type 1	Type 4	Type 3

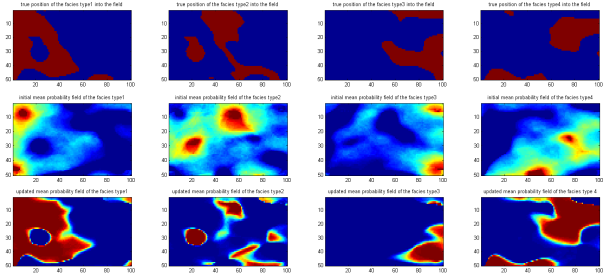
Model set up

- 120 ensemble members
- The GRF's used are generated with Gaussian variogram type, anisotropic with long length correlation of 30 gb, short length correlation of 15 gb and principal direction 0.
- 12 assimilation time steps of 20 days.

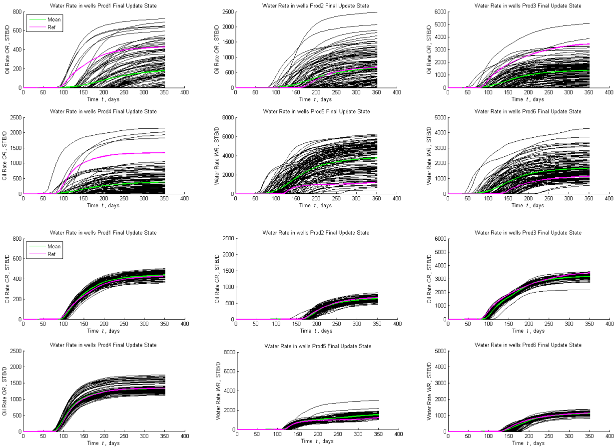
The measurement errors

- BHP: mean 0 and standard deviation 70 psi (1%)
- WR and OR: mean 0 and standard deviation 20 STB/D (1%)

Initial and updated probability maps



The Water rates



The geological consistency:The facies proportions

- The true facies proportions are 0.34:0.24:0.22:0.22
- The expected facies proportions (mean): 0.41:0.17:0.11:0.31
- The standard deviation: 0.014:0.021:0.02:0.017
- The minimum values : 0.37:0.12:0.05:0.25
- The maximum values : 0.44:0.24:0.17:0.34

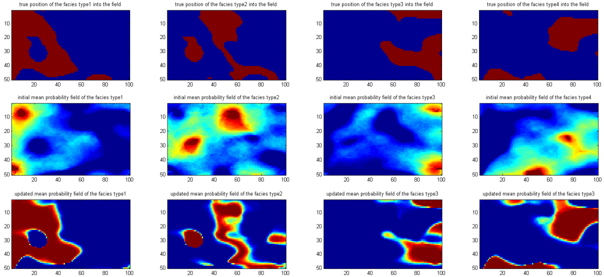
An extra control

The state vector for the j^{th} ensemble member at the k^{th} assimilation step :

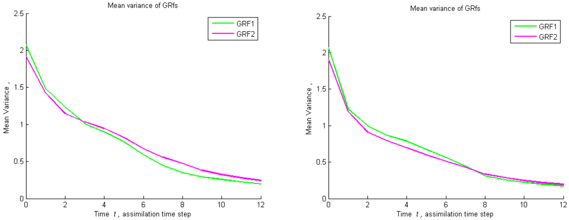
$$\mathbf{x}_j^k = \left[y_1^T \ y_2^T \ \mathbf{d}_{sim}^T \right]_j^\top , \quad (7)$$

where, $\mathbf{d}_{sim}$ are the simulated observations represented by the simulated production data (oil and water rates, bottom hole pressures) plus the facies proportions.
The observed facies proportion used 0.35:0.25:0.2:0.2 with 0.05 standard deviation measurement error.

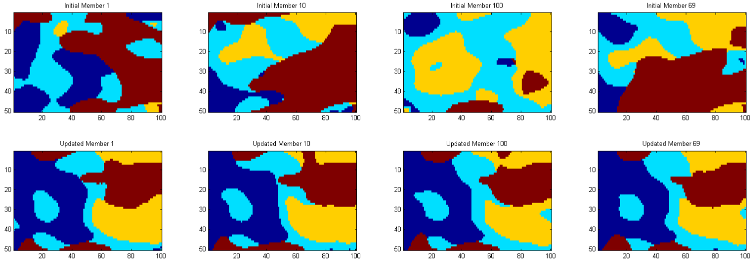
Initial and updated probability maps



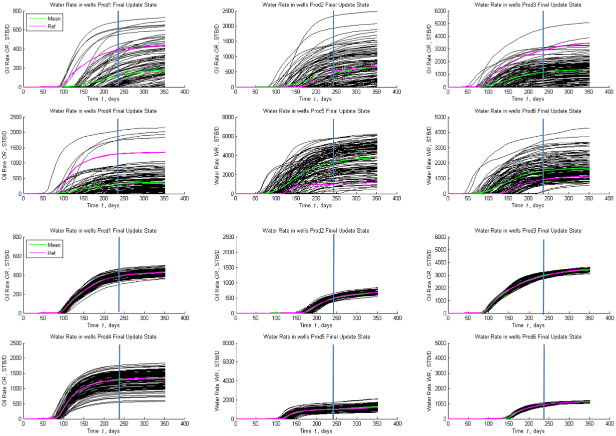
The variability reduction



4 Members


Initial Member 1
Initial Member 10
Initial Member 100
Initial Member 69
Updated Member 1
Updated Member 10
Updated Member 100
Updated Member 69
10
20
30
40
50
20
40
60
80
100

Water rates


Water Rate in wells Prod1 Final Update State
Water Rate in wells Prod2 Final Update State
Water Rate in wells Prod3 Final Update State
Water Rate in wells Prod4 Final Update State
Water Rate in wells Prod5 Final Update State
Water Rate in wells Prod6 Final Update State
Mean
Ref
Oil Rate OR, STB/D
Water Rate WR, STB/D
Time t, days
Water Rate in wells Prod1 Final Update State
Water Rate in wells Prod2 Final Update State
Water Rate in wells Prod3 Final Update State
Water Rate in wells Prod4 Final Update State
Water Rate in wells Prod5 Final Update State
Water Rate in wells Prod6 Final Update State
Mean
Ref
Oil Rate OR, STB/D
Water Rate WR, STB/D
Time t, days

The geological consistency:The facies proportions

- The true facies proportions are 0.34:0.24:0.22:0.22
- The expected facies proportions (mean):0.33:0.226:0.215:0.228
- The standard deviation: 0.01:0.013:0.012:0.012
- The minimum values : 0.30:0.19:0.18:0.20
- The maximum values :0.35:0.26:0.24:0.27

Conclusions

1. Model that links the experts work from the exploration phase of reservoir description with HM.
2. It's easy to be implemented, working for as many facies types.
3. Could be implemented for the case where core observations are not presents, only probabilities maps obtained from seismic interpretations.